

# **TORSIONAL VIBRATION MODELLING AND ANALYSIS CONTINUED**

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## **ABSTRACT**

The purpose for conducting a torsional study of a proposed design is to predict torsional natural frequencies and forced response amplitudes caused by dynamic torques that are generated by the driven and/or driving equipment. If the mean, or constant response plus that due to torsional vibration is predicted to exceed a safe level, then changes are proposed to ensure reliable performance.

The validity of the predictions is strongly dependent upon the accuracy of the mass-elastic properties that are used in the model. The modeling technique should account for the fact that there is always some level of uncertainty about these properties. A closely related concern is that the actual installation may differ from the drawings and other details supplied for the study; some such deviations can significantly affect the torsional response.

This paper discusses these effects and describes a methodology for handling the uncertainties. Typical levels of uncertainty in predicted torsional natural frequencies are presented. Field results are compared to predictions. Real world examples show what can happen when the model and the machine differ in torsional properties.

## **1. INTRODUCTION**

A design-based model should represent the actual system well enough to make engineering decisions about design changes. However, the mass elastic data used to create the model have tolerances associated with them. This paper 1) maintains that the tolerances for input data must be considered in a reliable design process and 2) demonstrates one method for achieving this.

Case histories are presented which demonstrate the necessity for accurate input data and the consideration of tolerance bands.

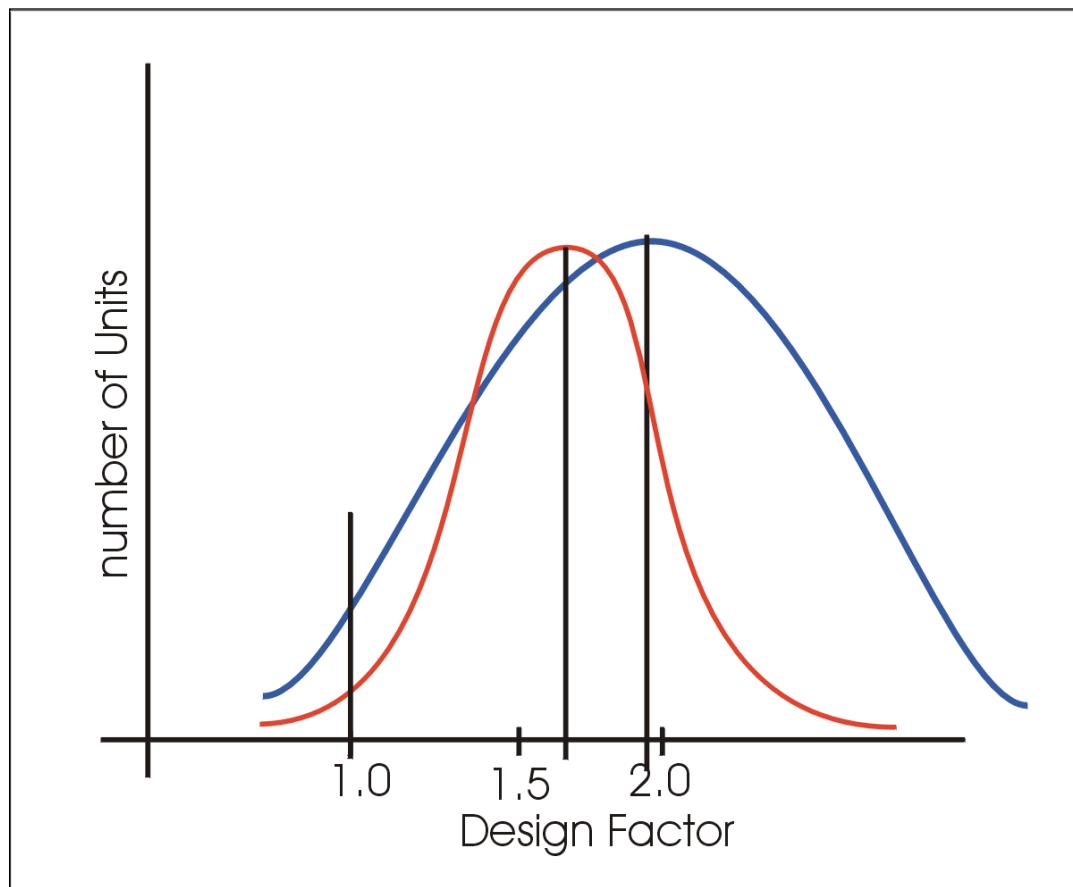
## **2. TOLERANCE BANDS**

Without considering a tolerance band for design input data, two possibilities exist which lead to design errors:

- 1) With traditional design processes, it is necessary to ensure calculated design factors are close to 2.0, as anything much less than 2.0 may compromise the reliability of a system. Because design factors significantly higher than 2.0 most likely ensure that the system will be satisfactory regardless of a reasonable uncertainty in input, expensive and/or unnecessary modifications may be recommended.
- 2) The actual system is not well represented and a poor design is mistaken for acceptable. This could be the case where mass-elastic tolerances lead to resonance and nominal values yield an acceptable margin of separation from resonance.

Essentially the design process must consider the statistical distribution of the actual safety factor. The actual safety factor is the ratio of actual stress to allowable stress. Both the allowable stress and the actual stress will be different from what has been assumed in the design process; thereby making it necessary to consider a tolerance for input data to ensure a safe and economic design.

The distribution of actual safety factors for two different design processes is described in Figure 1. The blue line represents a system designed using traditional methods where the target is a design safety factor of 2.0. The red line represents a design which has considered tolerance bands. The number of units which will experience failures is represented by those where their actual safety factor falls below 1.0 (actual stress exceeds allowable stress.) An analysis that utilizes a tolerance band decreases the uncertainty in the design calculations, which reduces the width of the distribution curve. The expected number of failures is fewer for the design using a tolerance band analysis with a design factor of 1.7 than for the design using more traditional methods and a design factor of 1.9.



**Figure 1 : Probability of Failure for Traditional (Blue) and Tolerance Band (Red) Design Processes**

### 3. METHODOLOGY

Methods for conducting torsional studies vary from simple hand calculations, to tabular methods, to finite element modeling. A tolerance band can be applied to any of these methods.

Mass elastic information should be listed with nominal values and the assumed or manufacturers' tolerances. Calculations can then be completed with values at the nominal, and upper and lower bounds. The effect of this is illustrated in the following example of a two mass, one spring system:

$$J_1 = 75 \text{ lb-in-s}^2 \pm 2\%,$$
$$J_1 = 125 \text{ lb-in-s}^2 \pm 5\%,$$

And

$$K = 30 \times 10^6 \text{ lb-in/rad} \pm 10\%$$

To calculate the nominal torsional natural frequency (TNF) the designer must use the nominal values for the inertias and spring constant. The TNF for a two mass system is:

$$TNF = \frac{1}{2\pi} \sqrt{K \left( \frac{J_1 + J_2}{J_1 \times J_2} \right)}$$

The calculated nominal TNF is: 127 Hz

To calculate the lowest possible TNF, the designer uses the highest possible inertias and lowest possible spring constant.

The calculated "low" TNF is: 119 Hz

To calculate the highest possible TNF, the designer uses the lowest possible inertias and highest possible spring constant.

The calculated "high" TNF is: 136 Hz

Suppose the machinery is operating at 1185 RPM.

$$1185 \text{ RPM} = 19.75 \text{ Hz.}$$

$$127/19.75 = 6.4x - \text{ample separation from a resonant order}$$

$$119/19.75 = 6x - \text{resonant with the 6th order}$$

$$136/19.75 = 6.9x - \text{close to 7th order resonance. Some separation, but still not good to be this close to resonance.}$$

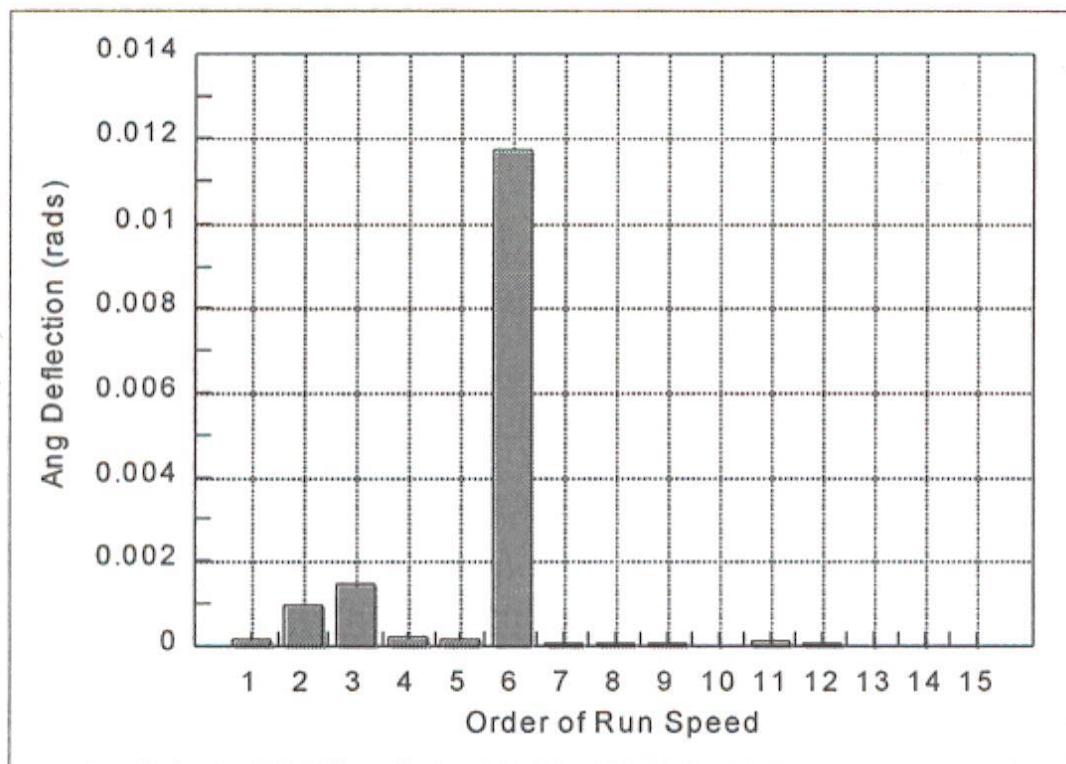
**The difference between the nominal and the upper and lower TNF values could mean the difference between a successful, reliable system and certain failure.**

#### 4. CASE HISTORY 1

Reciprocating compressor systems often include a lube oil pump driven from the opposite drive end (ODE) of the compressor shaft. This particular system was comprised of a

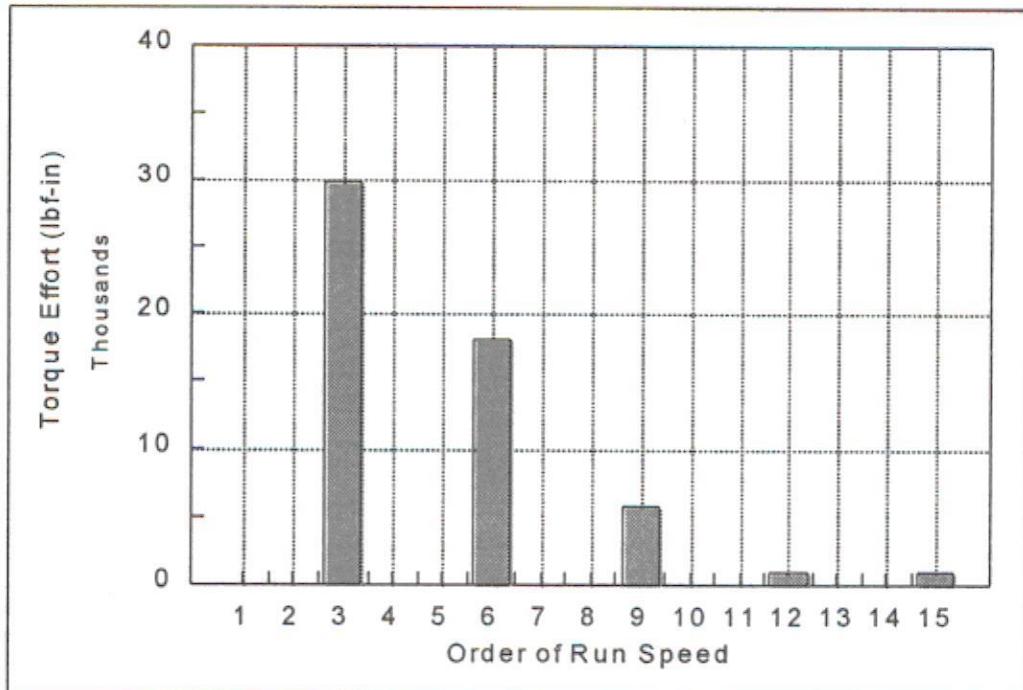
- six throw reciprocating compressor operating at 885 RPM – driving an auxiliary lube oil pump,
- a flexible disc coupling, and a
- 3000 HP squirrel cage induction motor.

Torsional vibration caused the oil pump drive shaft to fail within one hour of service. The pump was subsequently replaced but again failed within one hour of service.



**Figure 1.1: Torsional response at the compressor ODE was excessive at the 6<sup>th</sup> order of run speed.**

A preliminary analysis indicated that the first torsional natural frequency (TNF) of 80.5 Hz (5.5x run speed) was appropriately positioned between fifth and sixth order resonances, and resulted in minimum dynamic torsional amplification. However as shown in Figure 1.1, the measured sixth order torsional response was excessive; more than 8 times greater than the third order response. Additionally, the sixth order torque effort (i.e. torsional excitation) was less than the third order torque effort (Figure 1.2). These facts lead to the conclusion that the first torsional natural frequency (TNF) was relatively close to the sixth order of run speed (88.5 Hz).



**Figure 1.2: Third order torque effort is greater than the 6<sup>th</sup> order torque effort (i.e. torsional excitation).**

Based on field run-down tests, the system's first TNF was determined to be approximately 87 Hz (5.9x run speed), 6.5 Hz greater than predicted. This discrepancy between the predicted and measured first TNF was the result of both erroneous system mass-elastic data and a larger than expected uncertainty band. That is:

- i) the coupling hub had been pushed onto the motor 6" beyond the end of the drive shaft, thereby reducing its effective length and significantly increasing the system torsional stiffness.
- ii) the coupling was torsionally stiffer than originally indicated, and
- iii) uncertainties in the motor mass-elastic characteristics were significant.

As well as quantifying the system mass-elastic and thereby system TNF uncertainties, the motor shaft length and coupling stiffness were corrected in the model. The theoretical model was effectively validated by the fact that the measured TNF fell within the newly calculated TNF uncertainty band. The theoretical model could then be confidently used to calculate the additional compressor inertia required to minimize interference of the first TNF with the sixth order torque effort.

Although the auxiliary oil pump is relatively insignificant in terms of the capital costs, it is a critical system component; an oil pump failure could bring an entire system "down". Auxiliary driven equipment such as lube oil pumps should be considered in a system torsional analysis.

## 5. CASE HISTORY 2

A torsional analysis was performed on a revamped unit. There was no time allotted for design changes prior to start-up. Design changes could be made after some time in operation. The unit consisted of a

- 4 throw, 2 stage reciprocating compressor, with a
- 2100 HP fixed speed induction motor operating at 885 RPM (replacing the original engine)
- and a flexible disk coupling

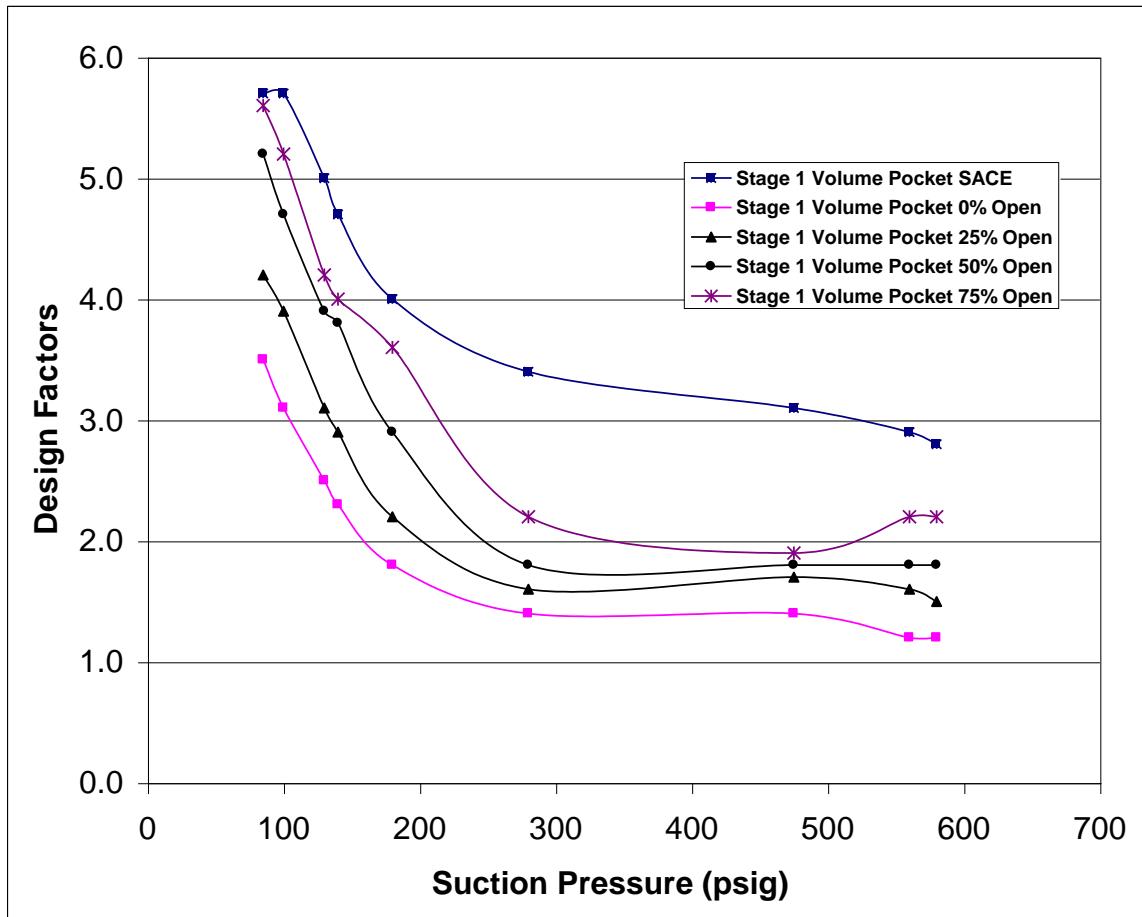
Additionally, the compressor had 2 active and 2 dummy cylinders. The 2 active cylinders were positioned closest to the coupling.

Predicted torsional natural frequencies (TNFs) are shown in the following table.

**Table 2.1: 1<sup>st</sup> Mode TNFs**

Scenario	1st_Mode_TNF	Order at 885 RPM
A - Low TNF	102.95	7.00
B - Nominal	105.89	7.18
C - High TNF	107.60	7.29

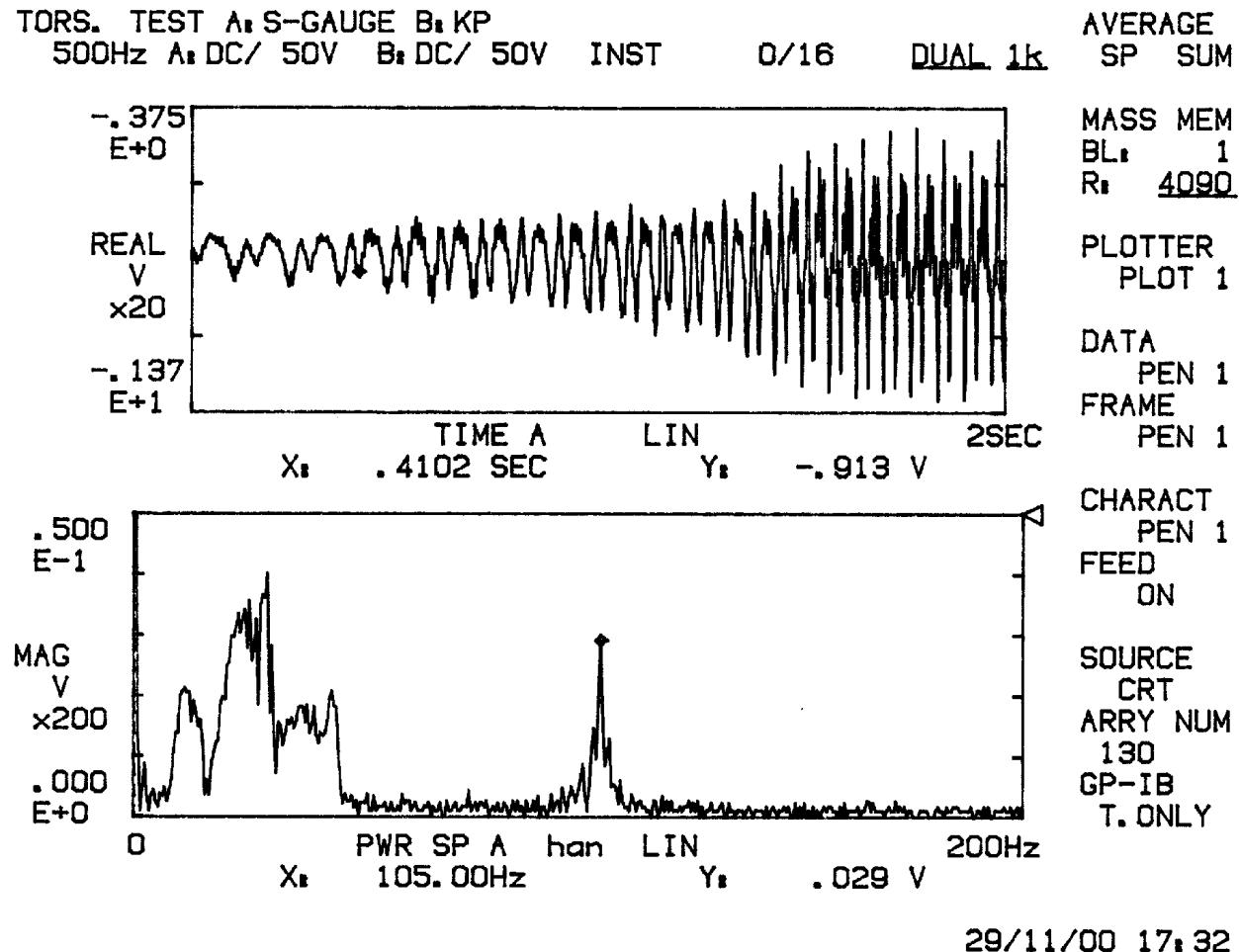
At 885 RPM, the Low and Nominal TNF cases were close to 7<sup>th</sup> order resonance. The motor drive stub was the weak element in the system due to a stress concentration factor in the keyway. Since the system was already installed and there was no time for changes, the customer wished to determine a range of conditions for safe operation. The torsional analysis was used to determine a recommended range of load steps for safe operation. These are illustrated in Figure 2.1 as drive stub design factors vs. load steps and pressures. Design factors close to 2.0 were acceptable.



**Figure 2.1: Drive Stub Design Factors at Various Load Steps - As Installed System**

The designer also recommended a field check to verify the predicted TNFs and torque levels. If the measured TNFs and torque levels were close to those predicted, lower design factors would be acceptable, thus widening the range of safe operating conditions.

A digital strain gage telemetry system was used to measure torques. A strain gage was applied at the motor drive stub, where predicted stresses were highest. TNFs measured during a startup are presented in Figure 2.2.



**Figure 2.2: Startup - Time and Frequency Plots showing 105 Hz Torsional Natural Frequency.**

The measured 1<sup>st</sup> TNF is 105 Hz, which is favorably in line with the Nominal TNF prediction – very close to 7X resonance. Measured torques at one of the load steps are presented in Figure 2.3.

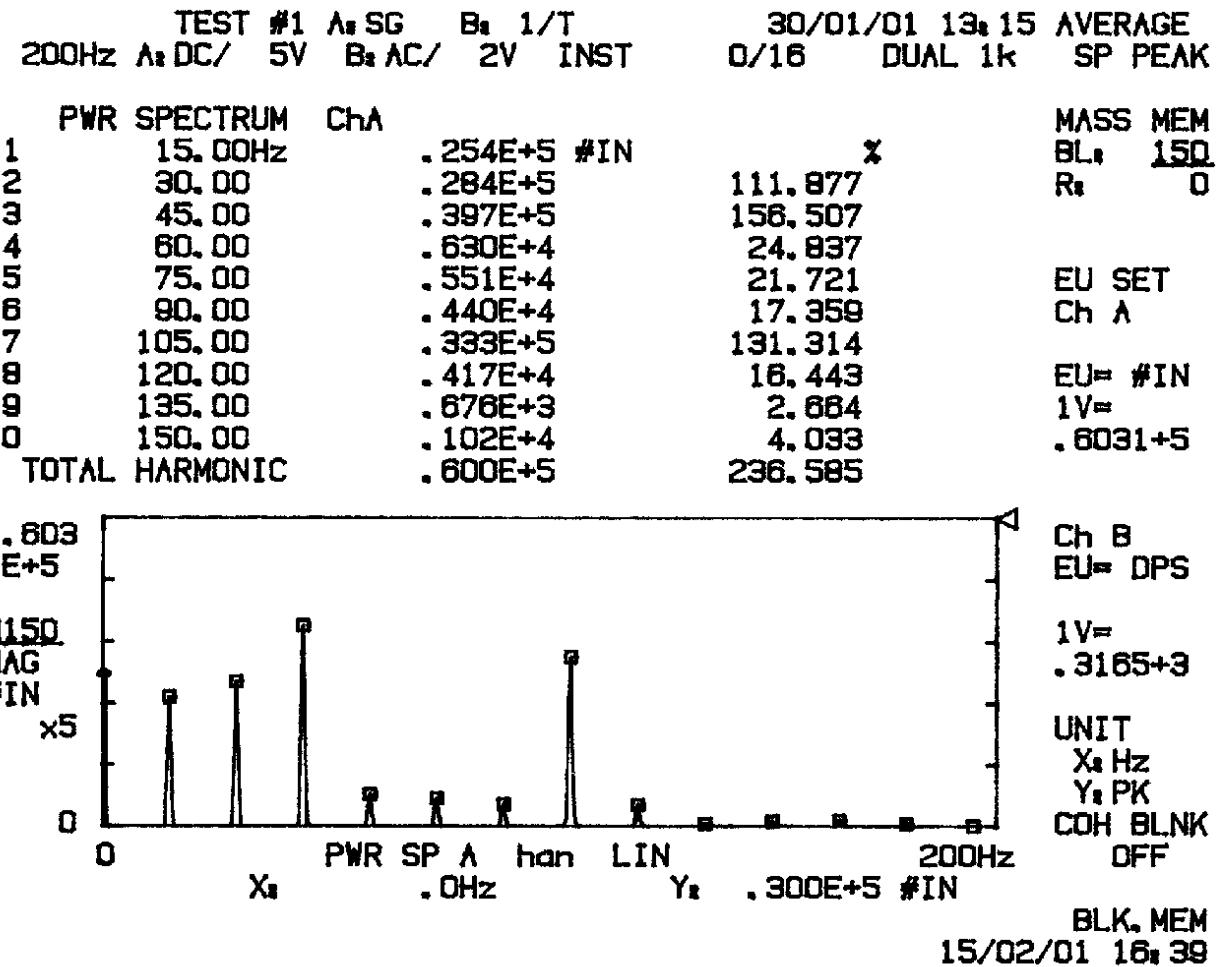
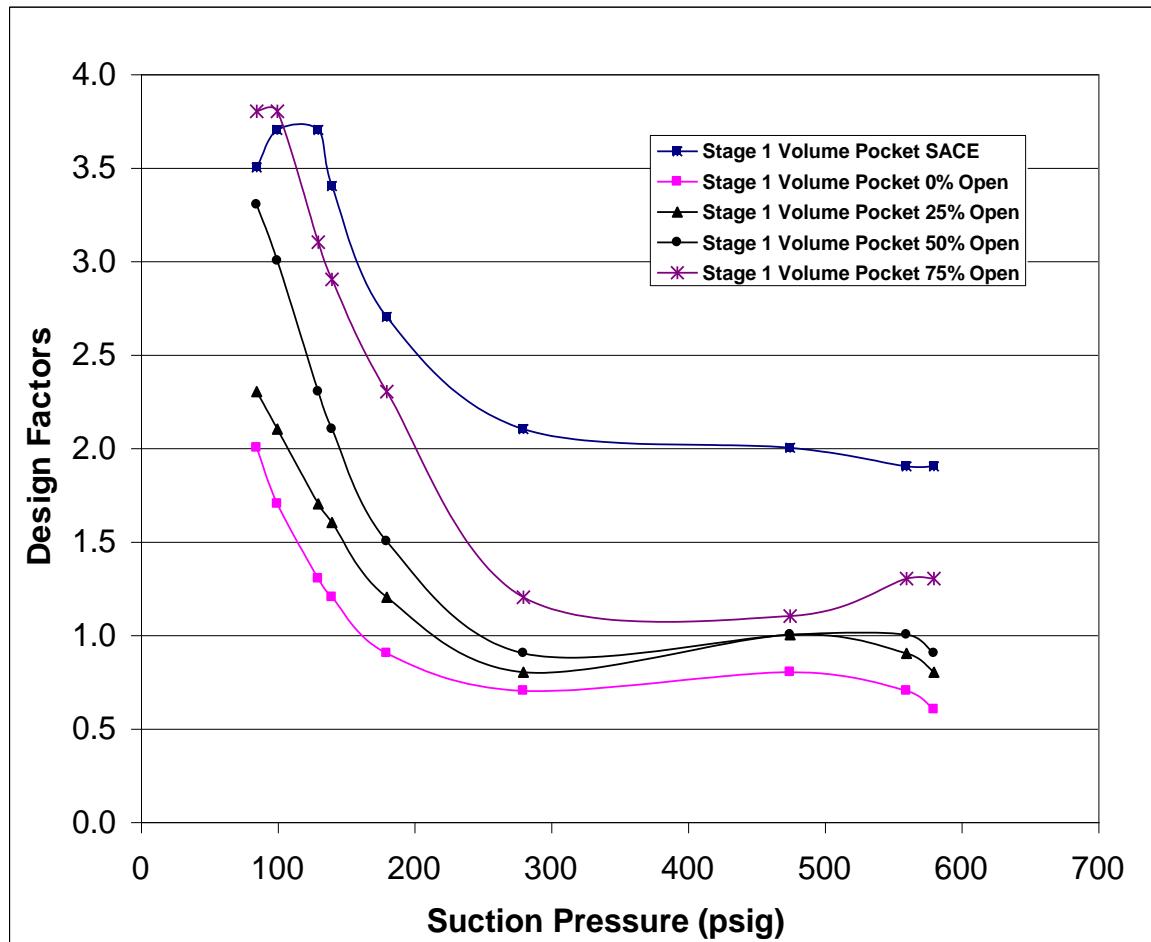


Figure 2.3: Torque Amplitudes Measured During Field Test.

These compare well with predicted torques when modal damping was reduced to match overall vibration levels. Note that the 7<sup>th</sup> order peak is significant, indicating a resonant amplitude.

The original model assumed a higher modal damping factor, but was reduced to obtain comparable overall vibration levels, since overall levels are used to determine stresses in the shafting. When torsional resonance is predicted, the system damping becomes a critical factor. This damping varies with frequency and machine particulars, many of which are difficult to quantify. In this case the subject unit was only using only 2 out of 4 throws. Lower friction (hysteresis) damping would therefore be expected.

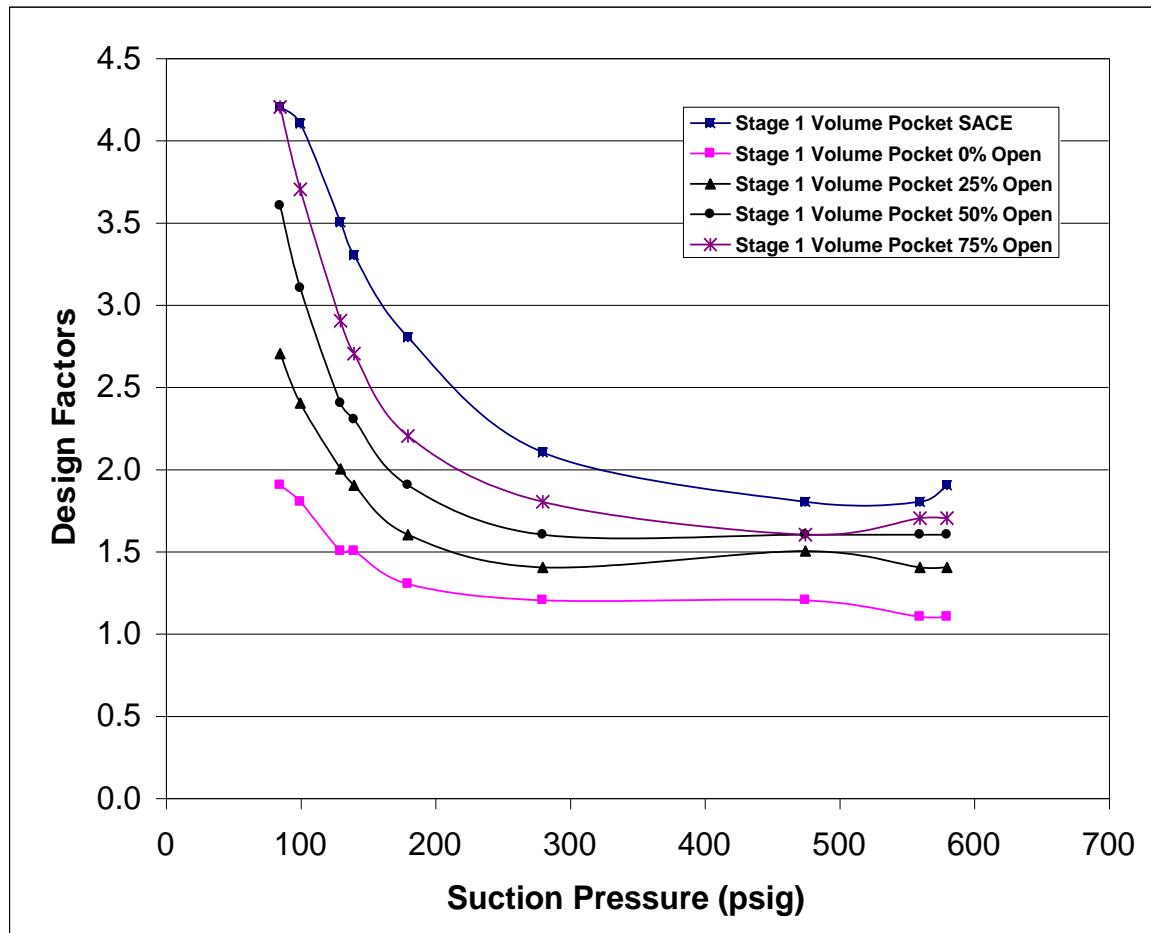
Having confidence in the predicted TNFs and torque levels, it was acceptable to specify load steps based on a lower design factor, since the width of the tolerance band had been narrowed by field measurements. The design consultant felt comfortable lowering the minimum design factor to 1.5. The new range of operating conditions is shown in Figure 2.4.



**Figure 2.4: Drive Stub Design Factors at Various Load Steps - As Installed System - 1% Modal Damping**

It was anticipated that the operating envelope could be widened. However, with damping reduced to match field data, the torsional stresses went up in the model. This actually served to narrow the envelope. Fortunately, the customer was still running the unit far under capacity and the new range of operating conditions would suffice for the time being.

More gas will be brought online in the near future. Therefore, the customer requires widening of the operating window. The only way to accomplish this is to detune the system. A change of coupling and addition of a flywheel is not a feasible choice, as a change of skid spacing would be necessary. However, a donut or compressor internal flywheel can be used as a detuner. Accordingly, a  $23,184 \text{ lb}_m \cdot \text{in}^2$  donut has been recommended. Confidence in the model suggests that the B case TNF is the most correct. Based on B design factors in the modified system, the window of operating conditions is given in Figure 2.5. Design factors of 1.5 or greater are acceptable.



**Figure 2.5: Drive Stub Design Factors at Various Load Steps - Modified System with 23,184 lb<sub>m</sub>·in<sup>2</sup> Donut – 1% Modal Damping**

With the addition of a donut, the operating envelope will be widened substantially from the system. Most of the load steps will be useable.

The use of a tolerance band was a significant advantage in the specification of operating conditions for this unit. The designer was successful at allowing the customer to run the unit safely under capacity and recommend a suitable modification to allow higher capacity when more gas is brought online.

## 6. CASE HISTORY 3

A torsional analysis was performed on a revamped unit. Changing couplings or flywheels meant changing skid spacing or ordering expensive custom fit couplings. The unit was comprised of a

- 4 throw compressor,
- a 1500 HP Induction Motor at 885 RPM, and
- a flexible disk coupling with a flywheel

This unit was reconfigured from one to two stage operation by replacing two 5-3/4" bore cylinders with two 10-1/4" bore cylinders. The total inertia went up slightly, but the 1<sup>st</sup> torsional natural frequency (TNF) remained virtually the same for both configurations, mostly due to the flywheel. Both one and two stage TNFs were very near 5th order resonance at the high TNF side of tolerance band, as shown in Table 3.1.

**Table 3.1: 1<sup>st</sup> Mode TNFs**

Scenario	1st_Mode_TNF	Order at 897 RPM
A	67.26	4.50
B	70.29	4.70
C	72.72	4.86

A TVA had been performed by others for the original one stage configuration. The predicted 1<sup>st</sup> TNF was 64.9 Hz.

The history of the unit during one stage operation suggested that either the system was not resonant or the resonant response was not high enough to cause failures, as there had been none reported. However, as shown in figures 3.1 and 3.2, the predicted torque effort was 33% higher during two stage than one stage operation.

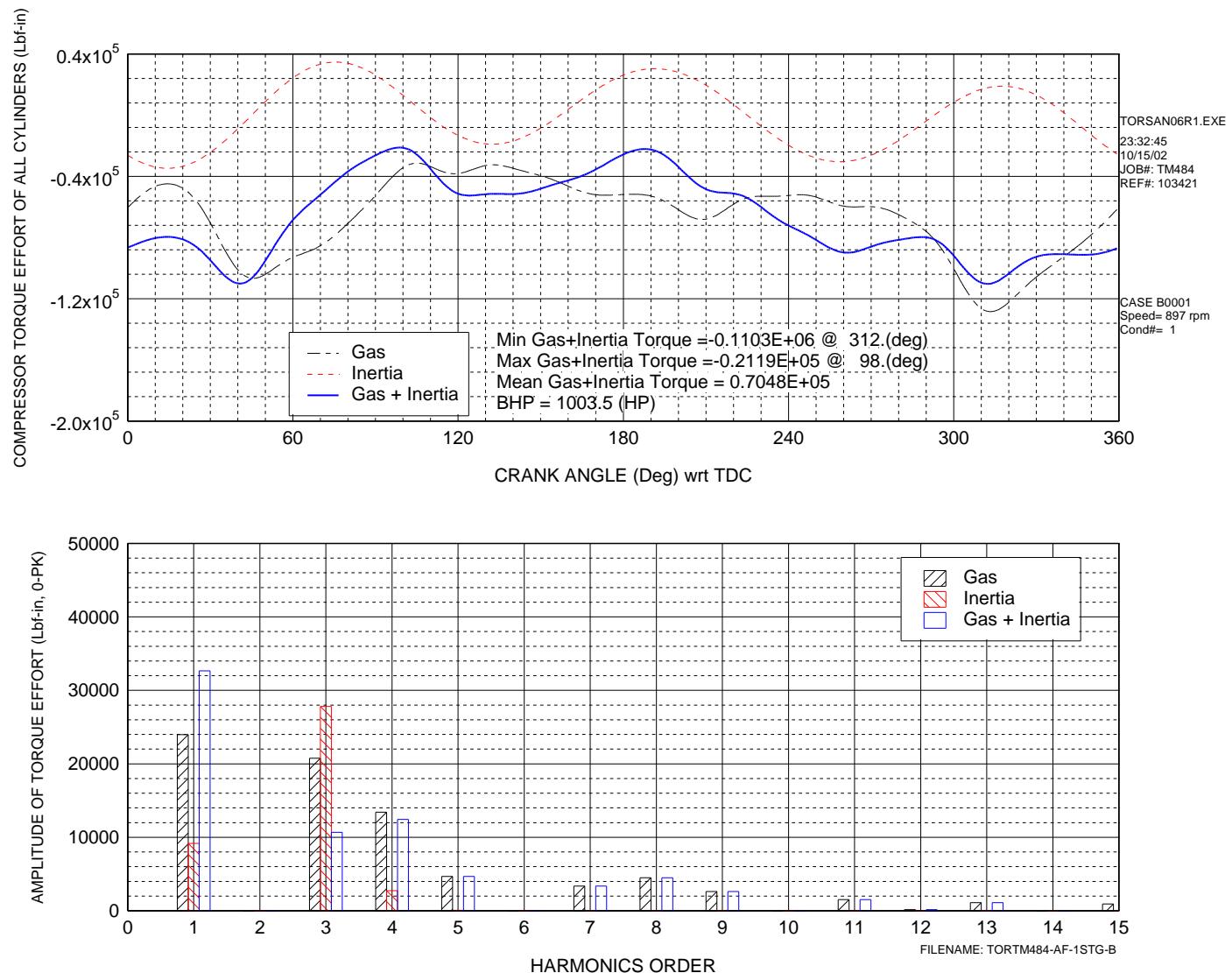
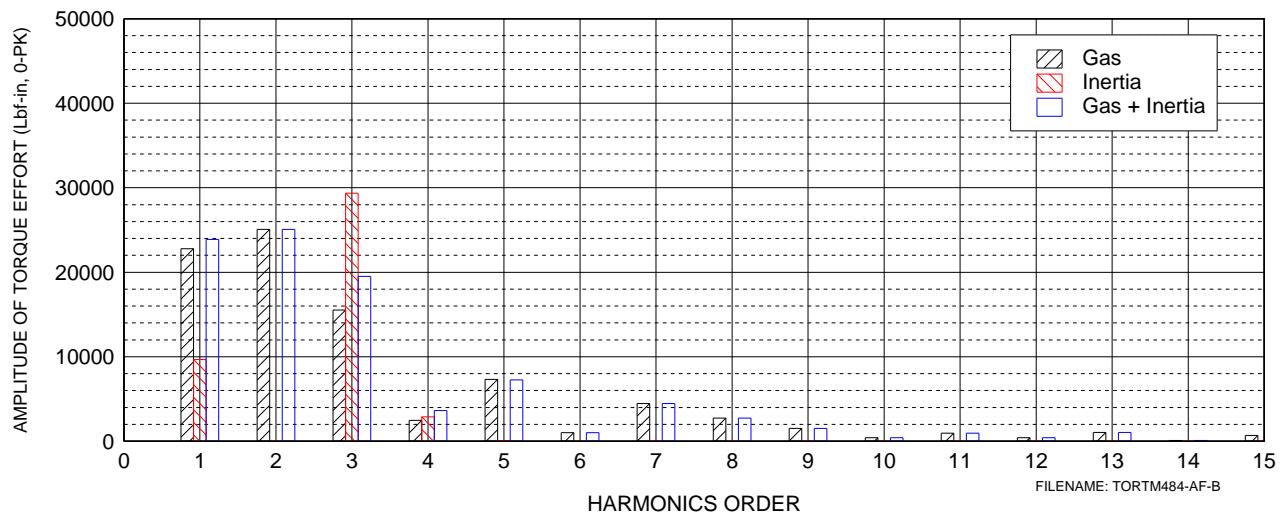
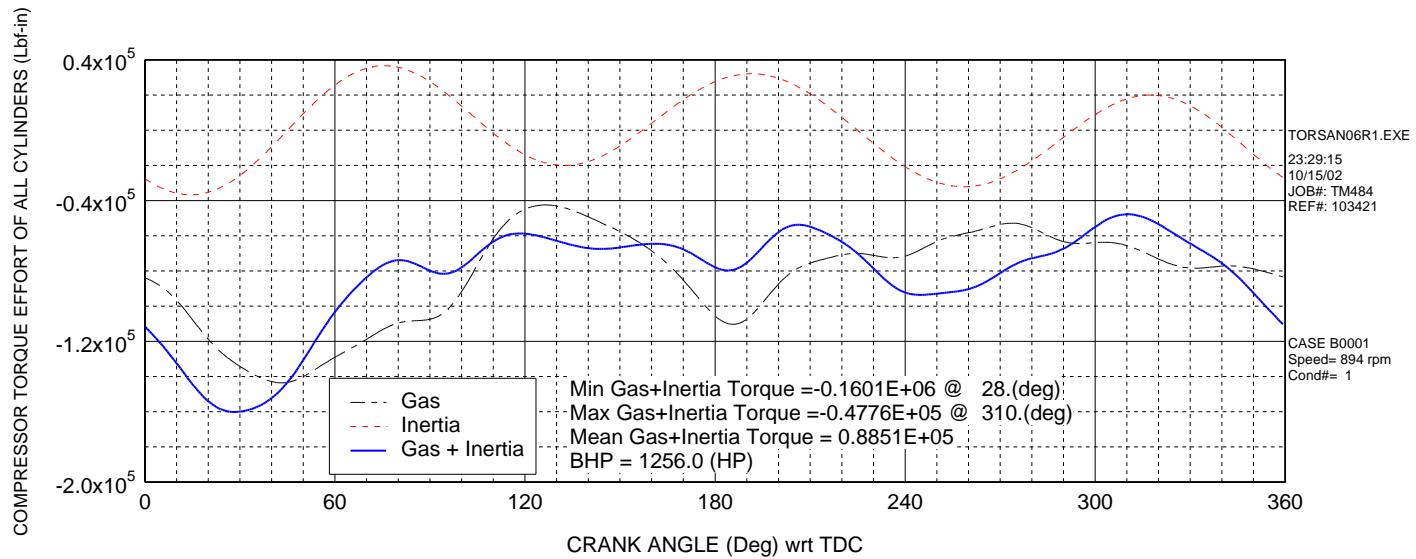


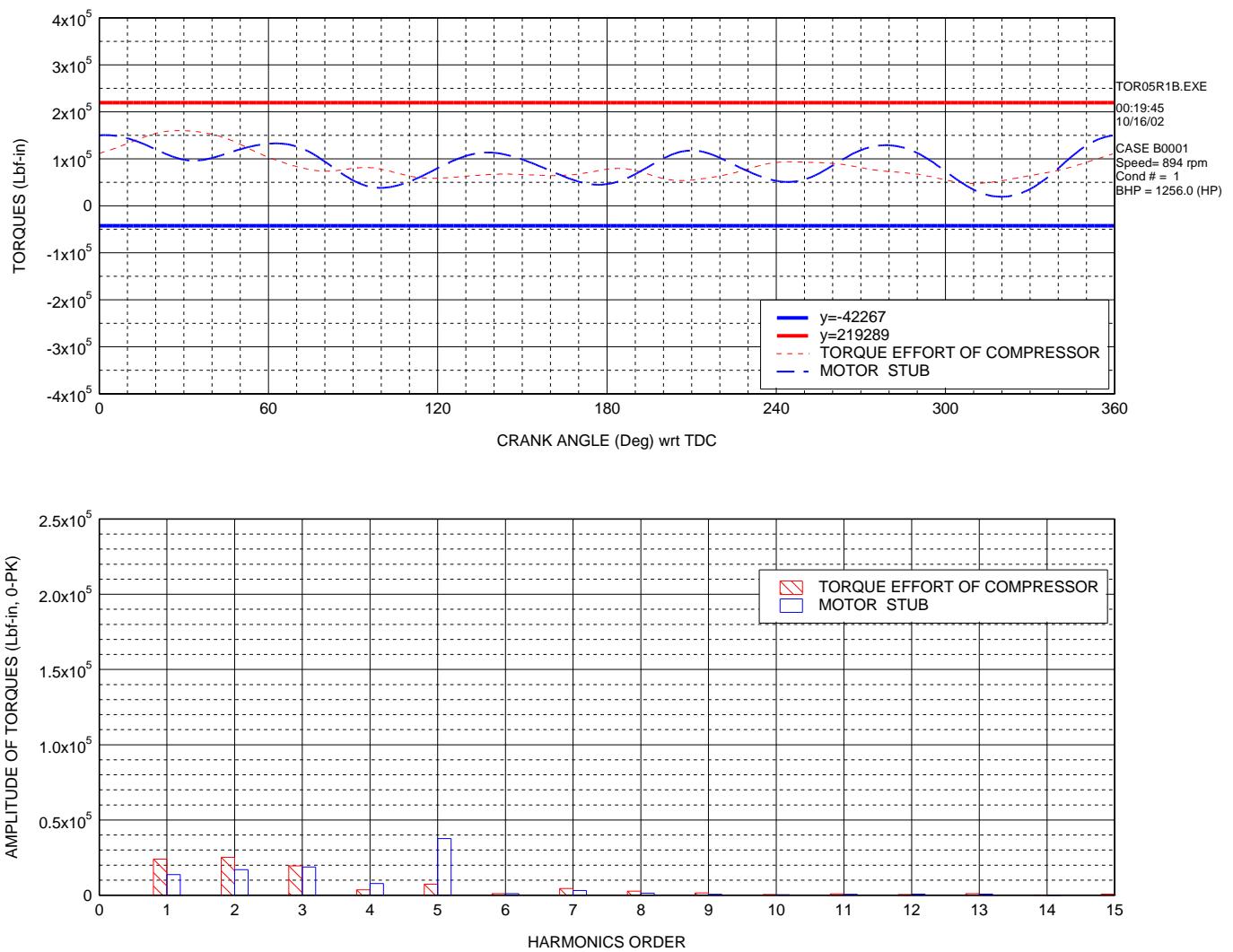
Figure 3.1: One stage torque effort - approximately 4200 lb<sub>f</sub>-in at 5x



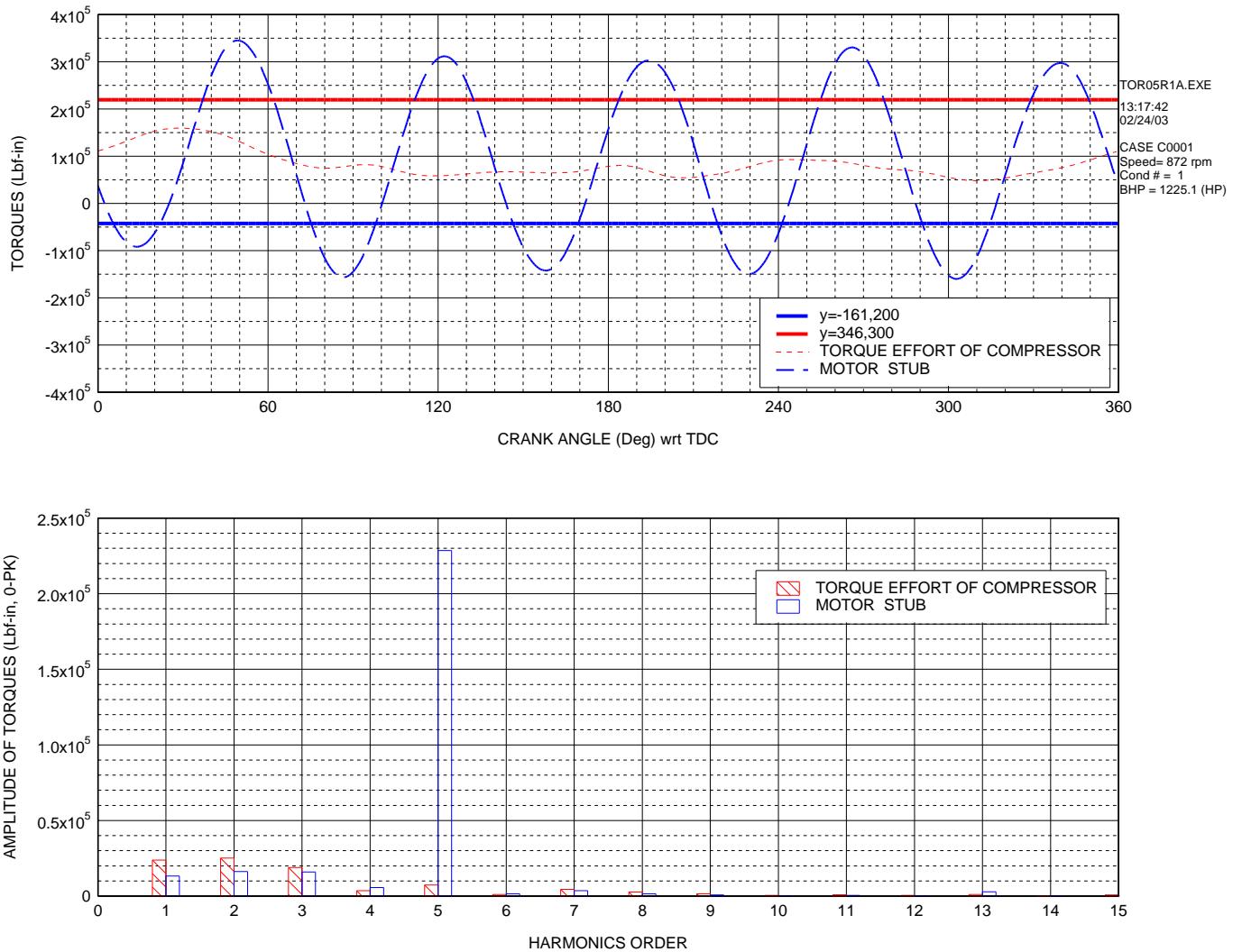
**Figure 3.2: Two stage torque effort - approximately 5600 lbf-in at 5x**

According to the new torque effort, the system could potentially have high enough response to lead to failure if it was in fact resonant.

The motor stub proved to be the weak element in the system. Away from resonance (B case response), the forced response yielded acceptable torques and stresses. However, a test of the system forced at resonance by adjusting the speed and damping indicated that the system was sensitive to 5<sup>th</sup> order resonance. The resonant response was 6 times higher than the response away from resonance. Magnification plots of the off and on resonance cases are shown in Figures 3.3 and 3.4 respectively.



**Figure 3.3: Two stage torque effort and response away from resonance (B case)**  
**- the blue and red lines are the acceptable upper and lower torque limits**



**Figure 3.4: Two stage torque effort and response at resonance (C case)**  
 $(5 \times \text{resonant speed} = 72.7 \text{ Hz} * 60 \text{ RPM/Hz} / 5 \times = 872 \text{ RPM})$   
- the blue and red lines are the acceptable upper and lower torque limits

These results led to the conclusion that even though the system may not be resonant, the risk was too high to leave the system as it was with higher 5<sup>th</sup> order torque effort during 2 stage operation.

An resonant system that is already installed can not easily be detuned by changing the flywheel and/or coupling because it often requires moving the driver or driven equipment on the skid or ordering couplings with custom made spacers spools. These options are usually expensive. Additionally, if the untested system wasn't resonant, making such a change could make it so. The most economic and reasonable option was to conduct a field check of the system to determine the location of the TNF. There was still time to check the system while it was in one stage operation. Since the TNF in two stage operation would be almost the same, the comparison between the two operation modes would be valid.

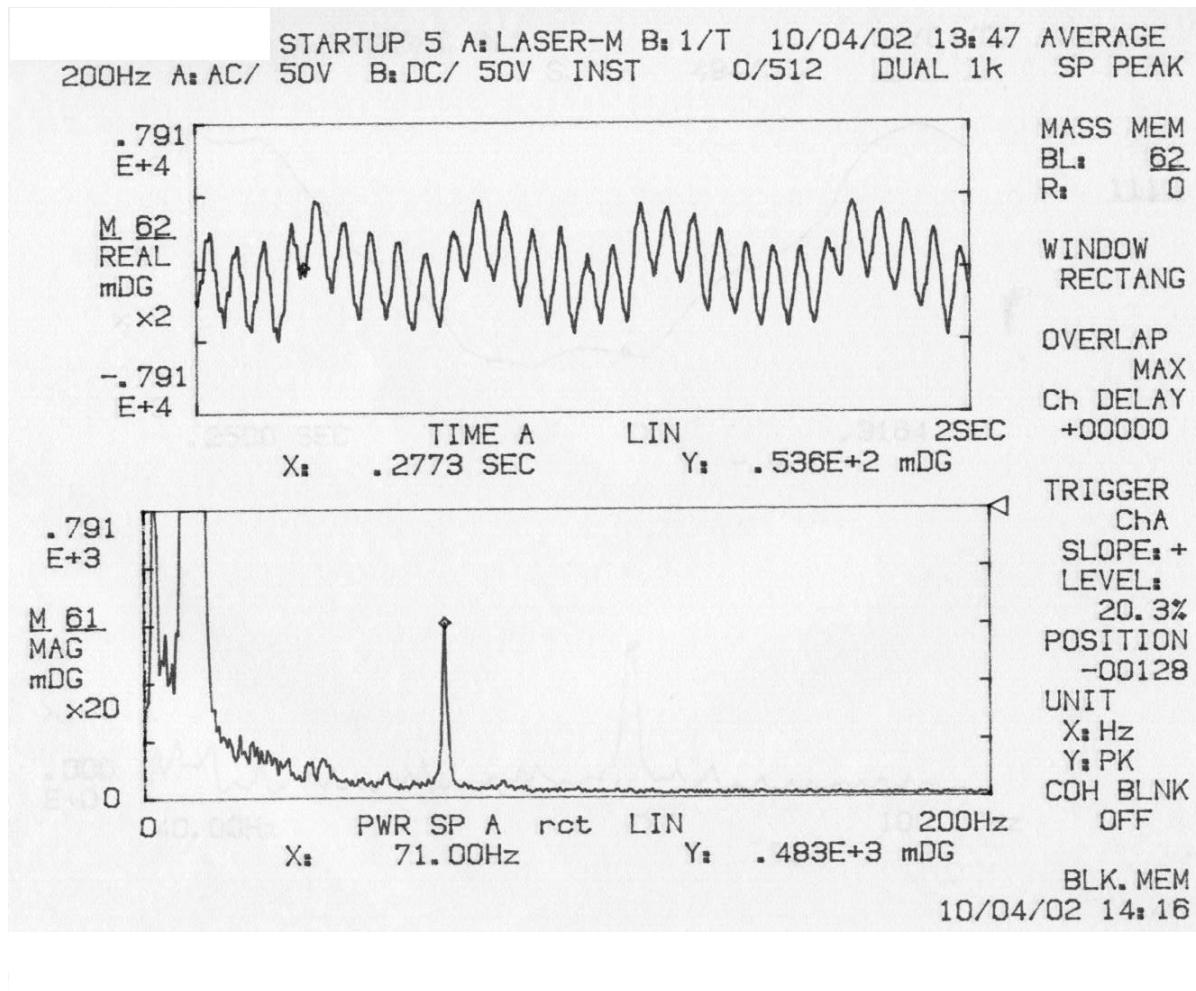
If the system wasn't resonant, no changes would be needed. The customer would only incur the cost of the field trip. If the system was resonant, then the response amplitudes could be

evaluated. There would still be a chance that they would be low enough not to warrant modifications. If modifications were in fact needed, then the optimum amount inertia or stiffness could be determined.

Accordingly, a field check was conducted. A torsional laser vibrometer was used to measure TNFs and torsional displacements - shown in Figure 3.5. The measured TNF is presented in Figure 3.6.

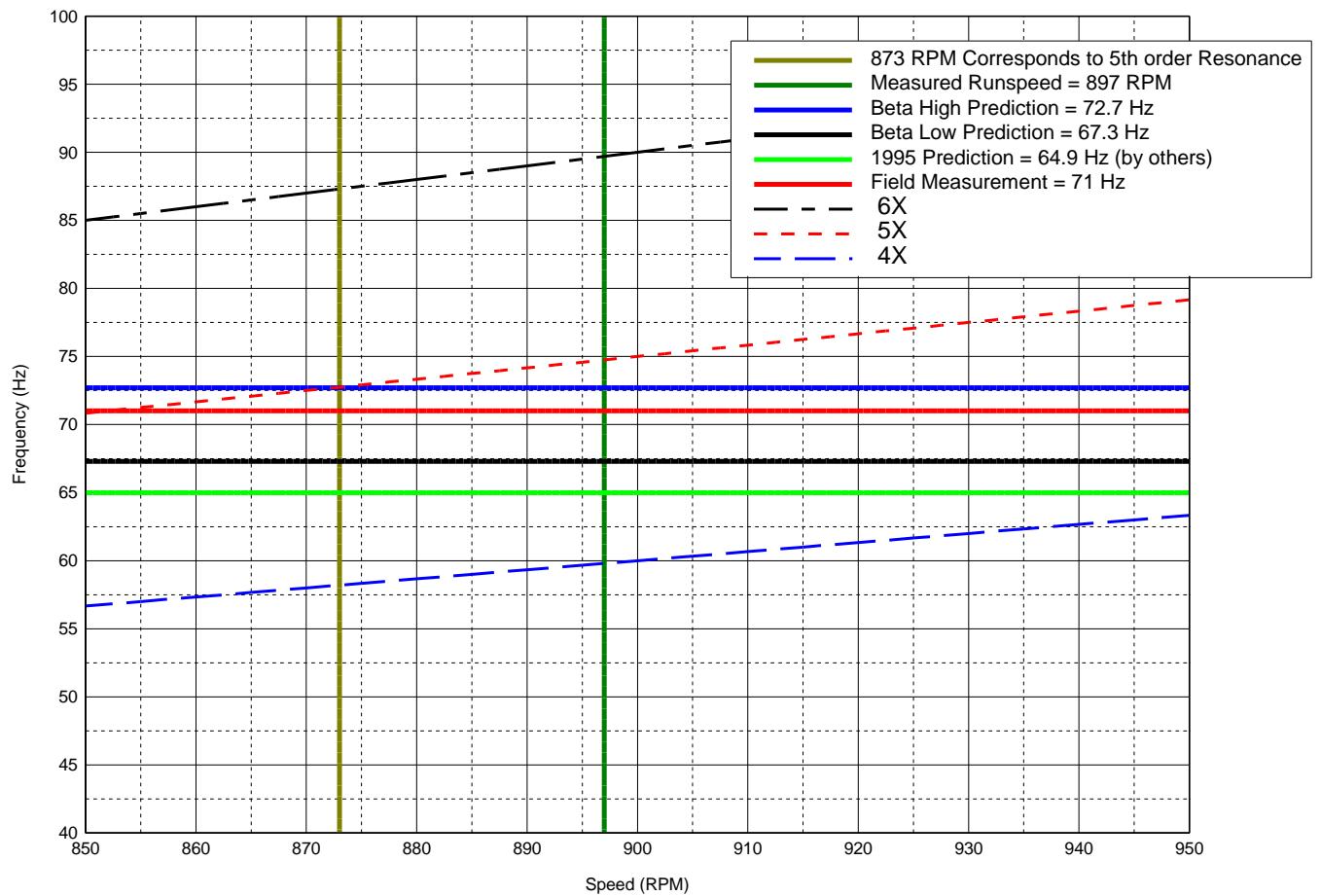


**Figure 3.5: Torsional laser vibrometer used to measure torsional vibration at the coupling**



**Figure 3.6: Measured TNF of 71 Hz**

The 71 Hz TNF agreed favorably with the prediction as it fell within the 67.3 to 72.7 Hz tolerance band, and was close to the nominal prediction of 70.5 Hz. The original analysis by others predicted the 1<sup>st</sup> TNF to be 64.9 Hz, below measured and outside of the 67.3 to 72.7 Hz band. An interference plot is shown in Figure 3.7. It summarizes the TNFs and speeds at which they correspond to resonance.



**Figure 3.7: Interference Diagram**

The measured TNF was well away from resonance and so the torques and stresses were sufficiently low. No modifications were needed and the field check was capital well spent.

The use of a tolerance band allowed the designer to check for sensitivity to resonance and also compare the new and old configurations. Unnecessary modifications were avoided and the owner of the unit saved significant capital.

## 7. CASE HISTORY 4

A torsional analysis was conducted on a newly installed unit to address reported oil pump and coupling failures . The system consisted of a

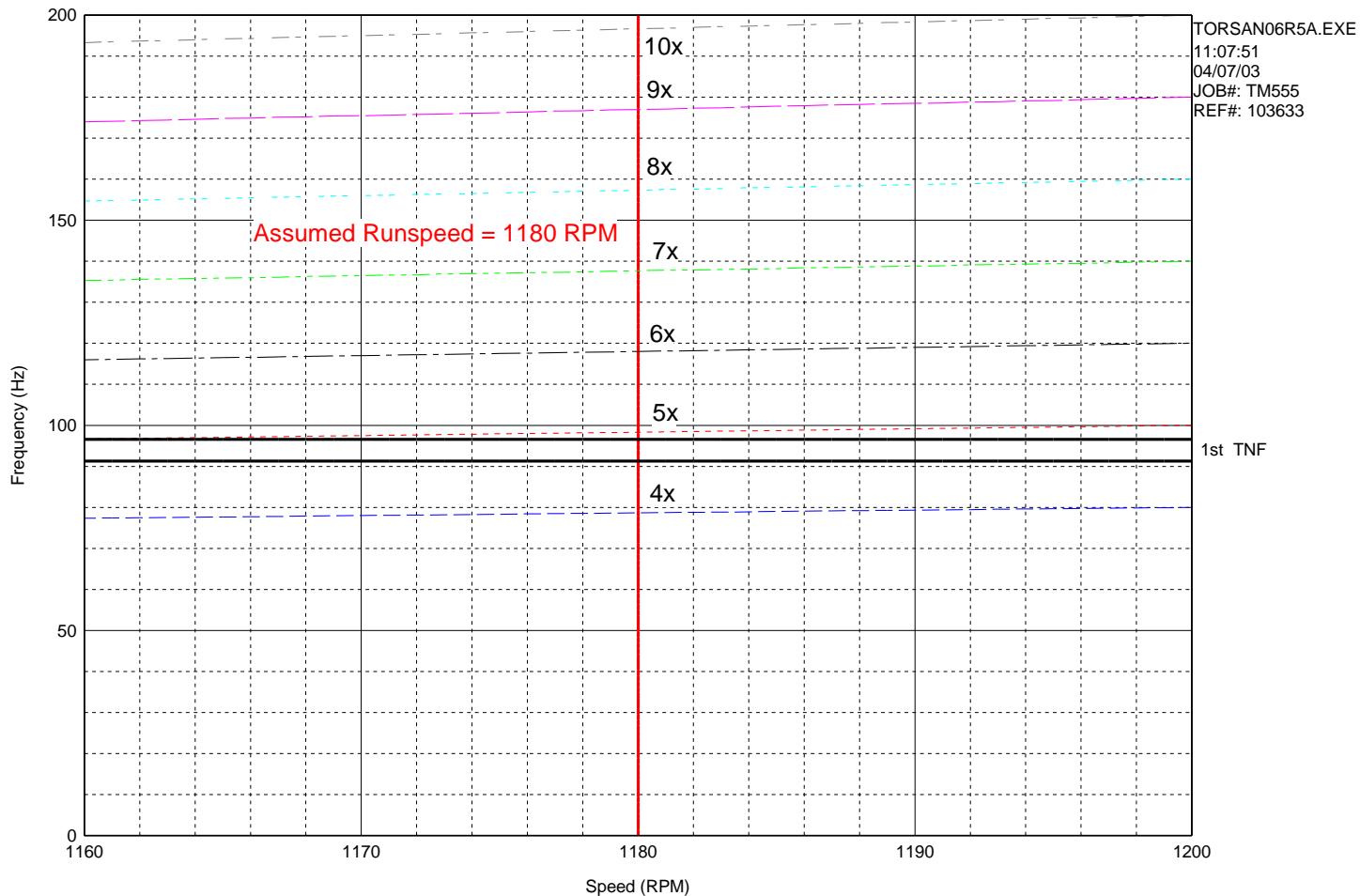
- 4 throw, 3 stage compressor,
- a 900 HP fixed speed induction motor at 1180 RPM,
- and a flexible disk coupling

Prior to the installation of the unit, a torsional analysis had been conducted by another service provider. Four 2500 lb<sub>m</sub>-in<sup>2</sup> crankshaft detuners (donuts) had been recommended and installed on the crankshaft spreader, centered evenly between throws.

The design analysis performed by the author considered the stiffening effect of the 2 piece clamped ring joint that makes up a donut. With this new stiffness added at the donut inertia locations, one of the predicted torsional natural frequencies (TNFs) was 96.6 Hz, which corresponds to 4.9x runspeed at 1180 RPM (96.6 Hz x 60/1180 RPM = 4.9x). The range of TNFs are shown in Table and Figure 4.1.

**Table 4.1: 1<sup>st</sup> Mode TNFs**

Scenario	#1_Mode_TNF	Order at 1180 RPM
A	91.26	4.64
B	94.43	4.80
C	96.60	4.91



**Figure 4.1: 1<sup>st</sup> Mode TNFs**

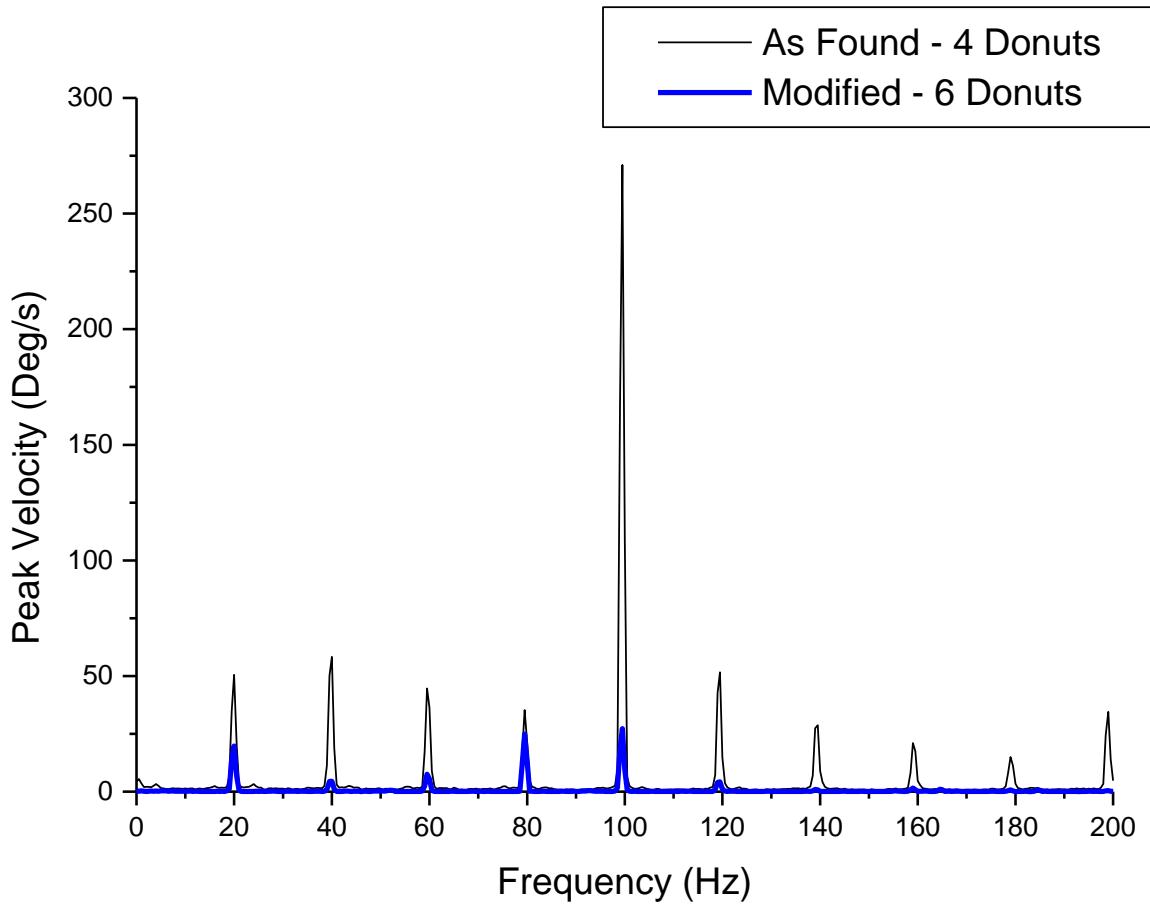
A forced response analysis was conducted with the speed and damping adjusted to test for sensitivity to 5x resonance ( $97 \text{ Hz} \times 60/5 = 1159 \text{ RPM}$  and reduced damping). The predicted coupling torques and oil pump end displacement amplitudes were over manufacturer's guidelines. This result correlated with the reported failures.

Additional donuts were recommended. Adding 2 donuts moved the predicted 1st torsional natural frequency more appropriately between 4<sup>th</sup> and 5<sup>th</sup> order torque effort. With this change, the predicted coupling torques and auxiliary end displacement amplitudes were lowered to acceptable levels.

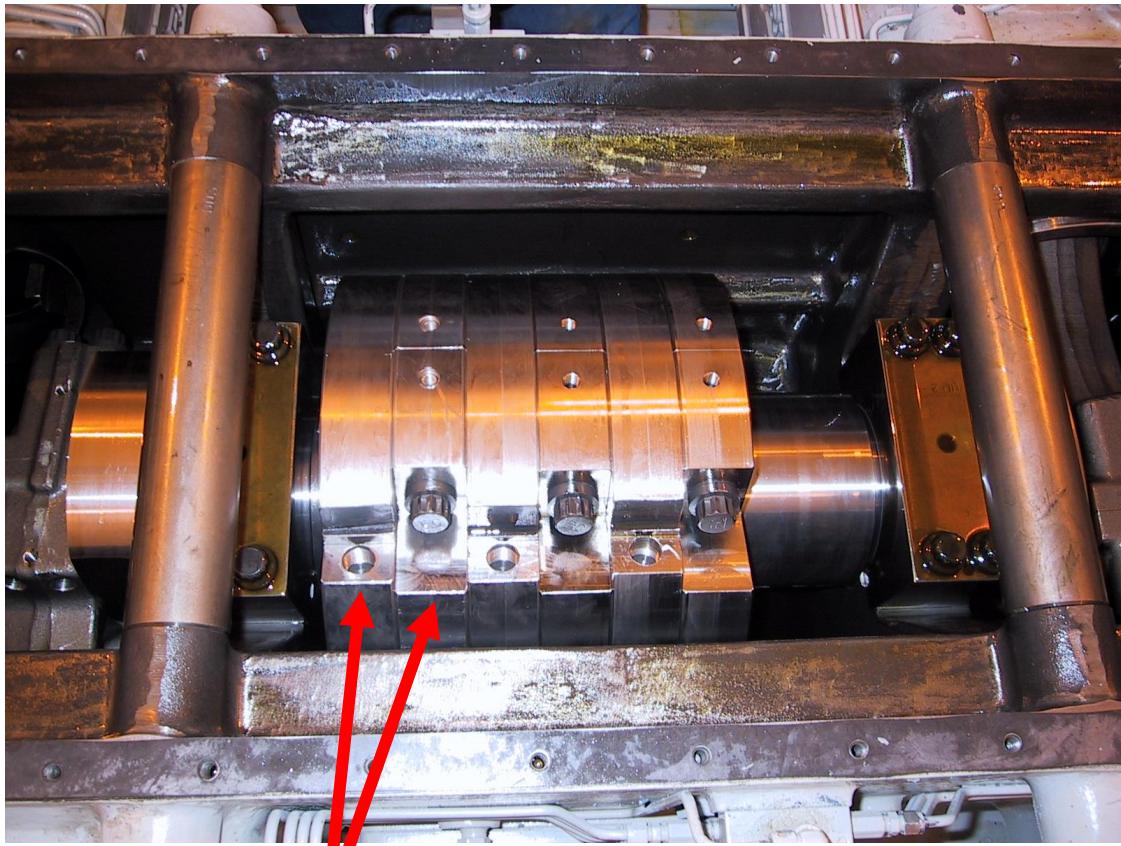
Two more crankshaft detuners were ordered by the end user. The donuts were to be installed only after initial field measurements could be made to verify torsional natural frequencies.

## Field Measurements

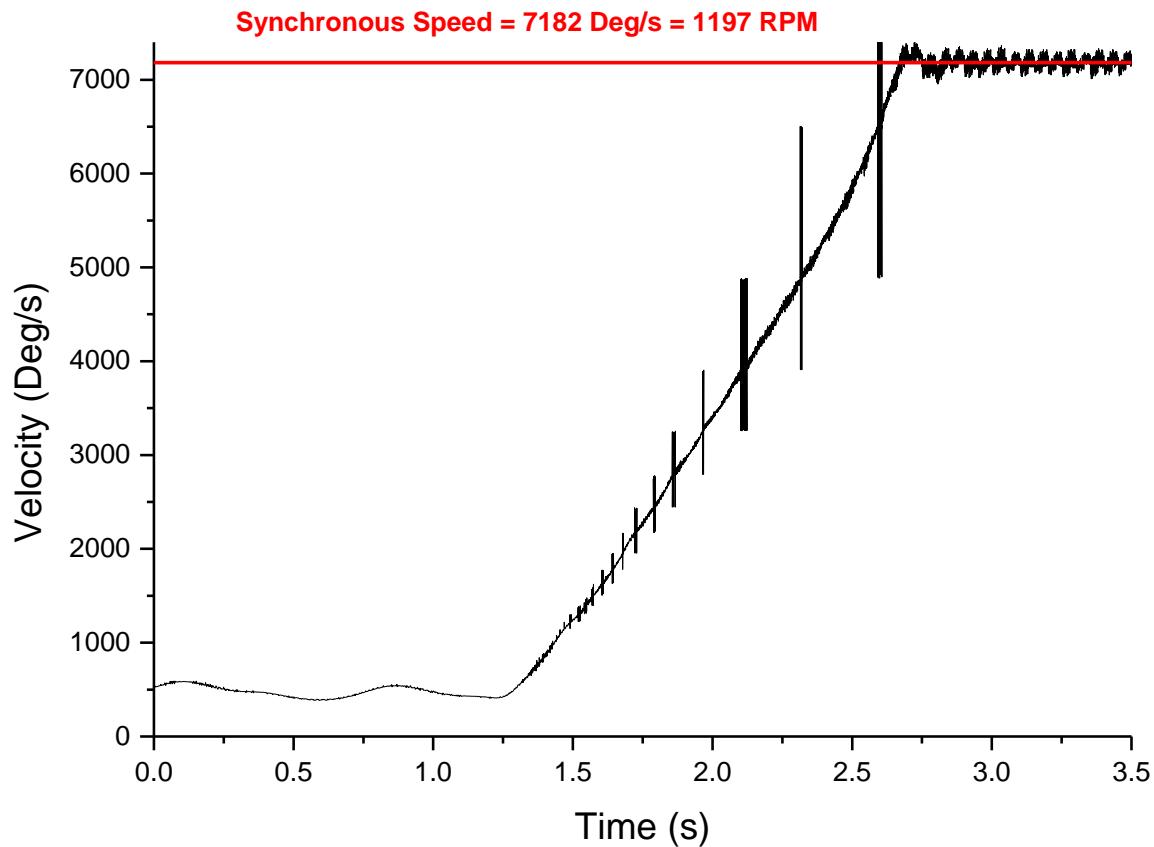
Spectrum plots of the as-found and modified systems at synchronous speed are overlaid in Figure 4.2. As predicted, the as found system indicated a strong 5<sup>th</sup> order resonance (high response near 100 Hz at synchronous speed in Figure 4.2). To move the 1<sup>st</sup> TNF as far away from 5x as possible, donuts were installed on the oil pump side of the existing 4 donuts - see Figure 4.3. The 1<sup>st</sup> TNF was thereby lowered to 93 Hz (4.7x at the actual runspeed of 1197 RPM) as shown in Figure 4.7. Figures 4.4 to 4.6 are the run up time data collected after the 2 extra donuts were installed. Measurements of the modified system show that the torsional vibration levels at the oil pump end of the compressor are much lower with 6 donuts than with 4 donuts - shown in Figure 4.2.



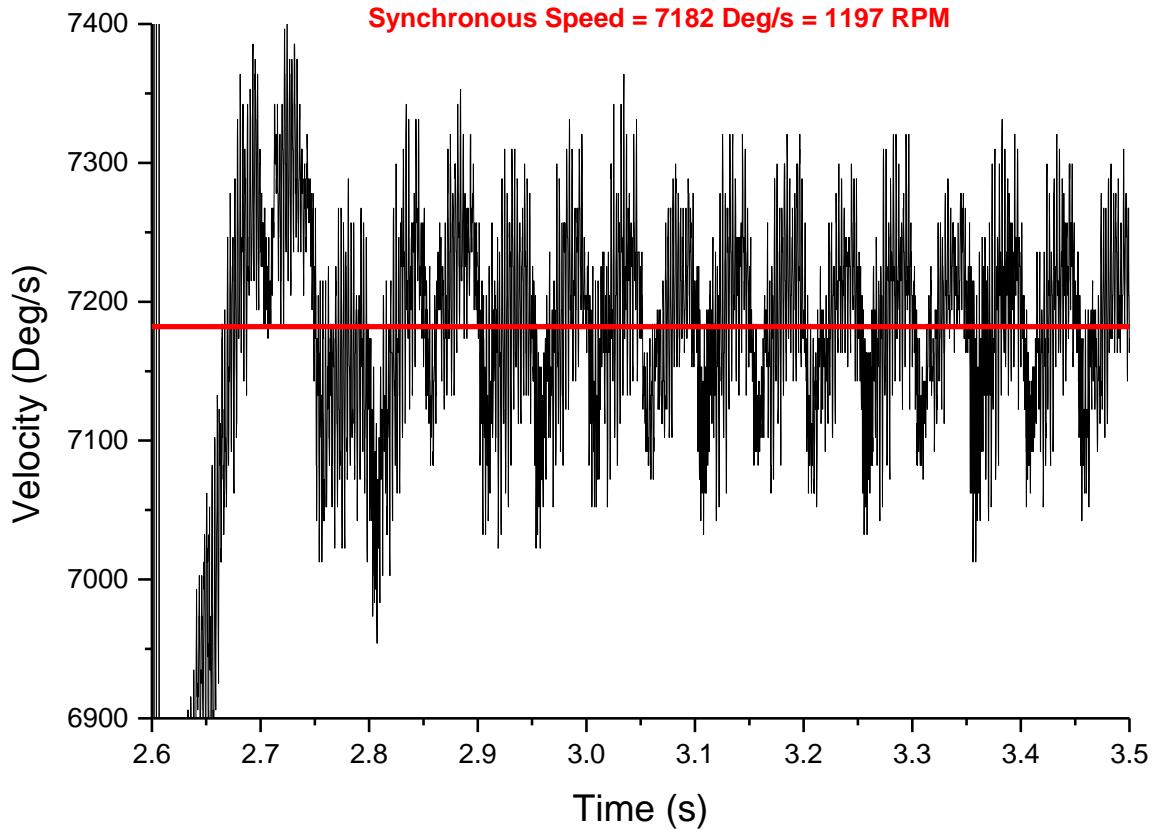
**Figure 4.2: As found and modified system torsional velocity at synchronous speed.**



**Figure 4.3: 6 Donuts - (2) added at oil pump end of existing 4 donuts**

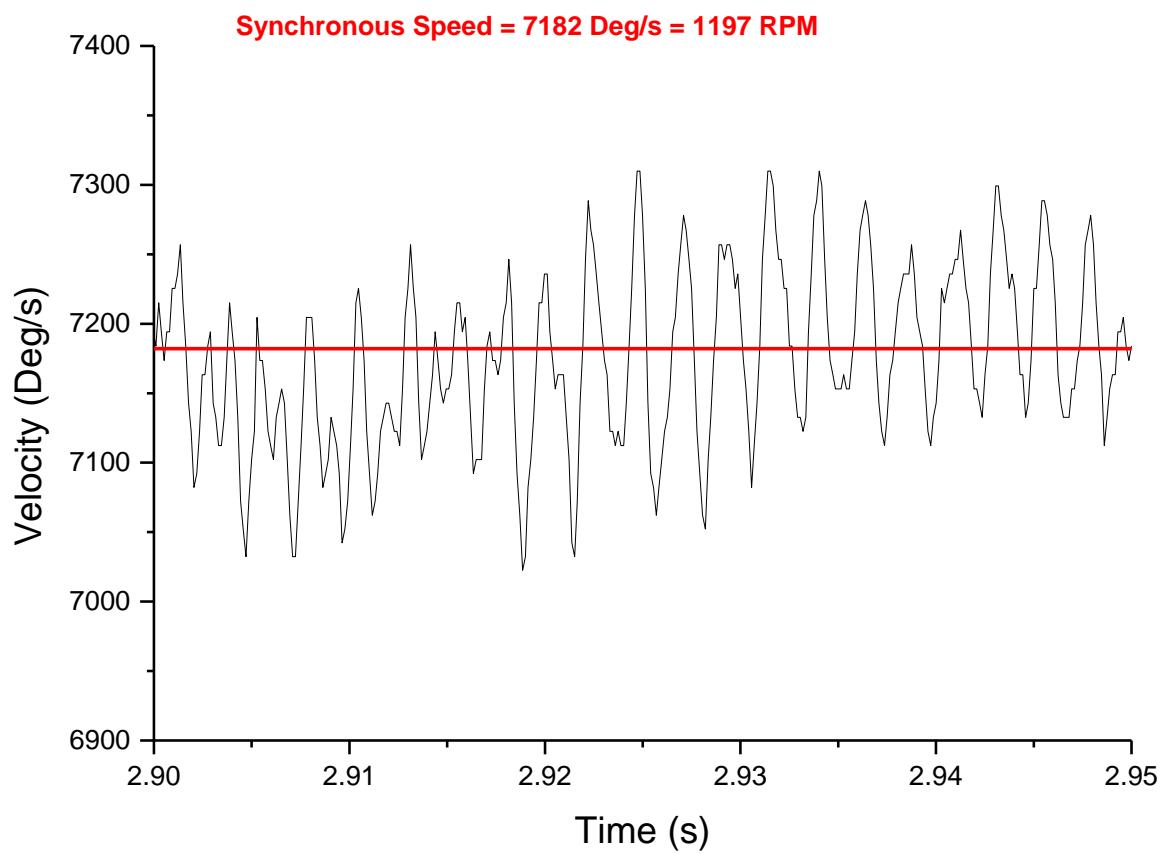


**Figure 4.4: Start-up ramp to synchronous speed with 6 donuts**

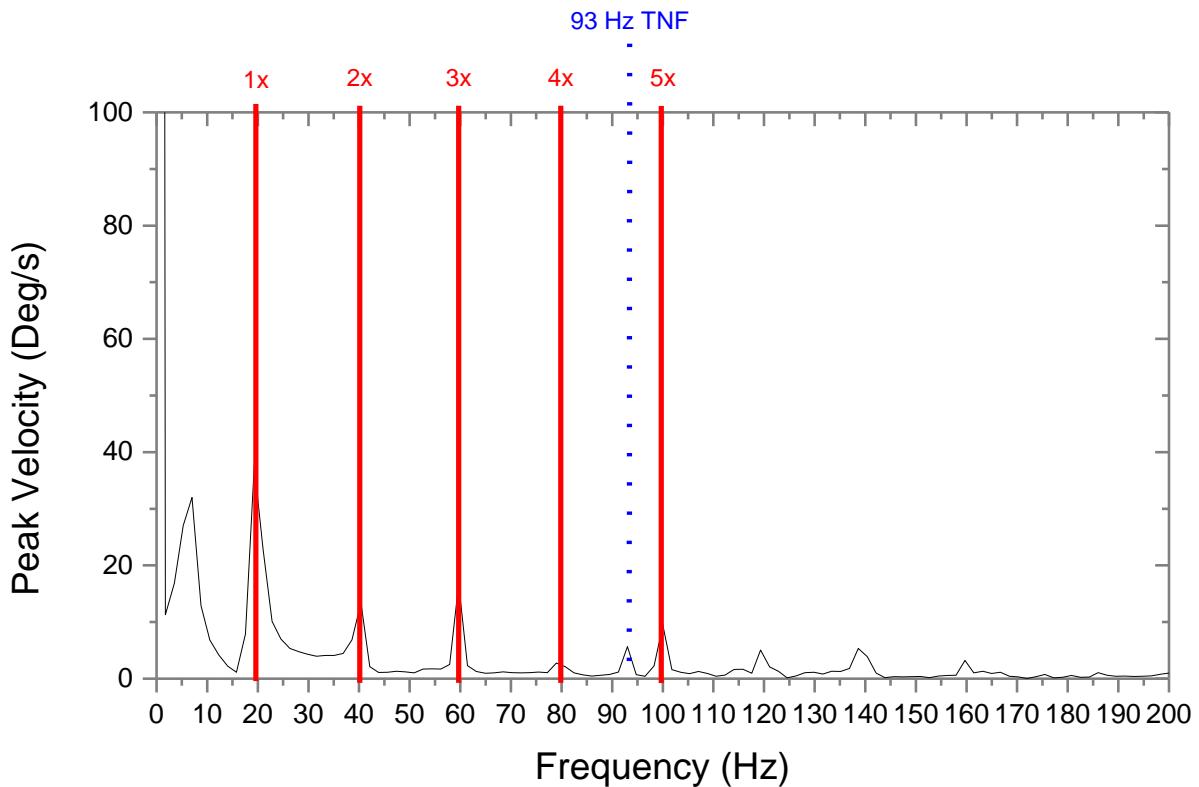


**Figure 4.5: End of ramp up to synchronous speed with 6 donuts**

The time window of 2.7 to 3.4 seconds in Figure 3.4 was chosen for the FFT operation. At the end of the ramp to synchronous speed, the system stops accelerating. The twist imparted to the system during the ramp up is released. This phenomenon effectively "rings" the system. It then oscillates at its torsional natural frequency until it is damped out. In Figure 4.5 the end of the acceleration ramp occurs at about 2.7 seconds. The torsional spring force of the system causes it to recoil beyond its mean twist position until about 2.8 seconds. Then from 2.8 to 3.4 seconds the oscillates at its TNF. One revolution of this torsional vibration is captured in Figure 4.6. The spectra is shown in Figure 4.7.



**Figure 4.6: One revolution at synchronous speed**



**Figure 4.7: Spectra of system with 6 donuts**  
 [from time window 2.7 to 3.4 seconds in Figure 4.5]

The nominal TNF prediction and corresponding forced response results did not show a problem with either the oil pump or the coupling. The use of a tolerance band allowed the designer to check for sensitivity to resonance at a higher predicted TNF and the results did correlate to the reported failures. The required number of crankshaft detuners were then determined but not installed until measurements could be made to verify the suspected resonance. Field results served to confirm resonance at the suspected harmonic order and verify the tolerance band used in the model. The modified system TNF also correlated well with the model. Most importantly, measured torsional vibration amplitudes were significantly lower after the donuts were installed. Problems with the oil pump and coupling were solved in a cost effective manner.

## **SUMMARY**

The mass elastic data used to create a torsional model has tolerances associated with it. These tolerances must be considered in a reliable design process. Without considering a tolerance band, design errors can occur. These errors can manifest themselves as an over or under designed system. An over-designed system is unnecessarily expensive. An under-designed system is unreliable and potentially unsafe.

Careful consideration must be given to the statistical distribution of the actual safety factor. The actual safety factor is the ratio of actual stress to allowable stress. Both the allowable stress and the actual stress will be different from what has been calculated in the design process; thereby making it necessary to consider the tolerance on input data to ensure a safe and economic design.

Field data can verify a design model, allowing for less conservative design measures. A successful design process should be modified to reflect historical field measurements.