Resonance Revealed: Understanding What Really Happens at Resonance

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RESONANCE

The word has various meanings in acoustics, chemistry, electronics, mechanics, even astronomy. But for vibration professionals, it is the definition from the field of mechanics that is of interest, and it is usually stated thus:

"The condition where a system or body is subjected to an oscillating force close to its natural frequency."

Yet this definition seems incomplete. It really only states the condition necessary for resonance to occur—telling us nothing of the condition itself. How does a system behave at resonance, and why? Why does the behavior change as it passes through resonance? Why does a system even have a natural frequency?

Of course, we can diagnose machinery vibration resonance problems without complete answers to these questions. But a fuller understanding can help lead us to the most effective solution, and naturally it is much more satisfying to have a real feel for what is happening. Whilst a little mathematics cannot be avoided, purely mathematical explanations can be a little too abstract for some readers. This paper therefore attempts to focus on some underlying principles and use these to construct vector diagrams to explain the resonance phenomenon. It thus aspires to provide a more intuitive understanding.

SYSTEM BEHAVIOR

Before we move on to the why and how, let us review the what—that is, what happens when a cyclic force, gradually increasing from zero frequency, is applied to a vibrating system.

Let us consider the shaft of some rotating machine. Rotor balancing is always performed to within a tolerance; there will always be some degree of residual unbalance, which will give rise to a rotating centrifugal force. Although the residual unbalance is due to a nonsymmetrical distribution of mass around the center of rotation, we can think of it as an equivalent "heavy spot" at some point on the rotor.

This heavy spot, and thus the centrifugal force, will complete a full cycle once per revolution. If there is little other excitation in the system, the displacement response, as measured in one radial plane, will be approximately sinusoidal.

At low speed, well below the system natural frequency, the peak or "high spot" of the vibration displacement cycle—

ABSTRACT

This article seeks to provide a more comprehensible explanation of the phenomenon of resonance using vector diagrams to describe how the spring, damping, and inertia forces balance to determine the behavior of a vibrating body when excited at frequencies below, at, and above natural frequency. Some hints on diagnosing resonance and a short case study are also provided.

KEYWORDS

resonance, natural frequency, phase, bode, coast down, displacement, velocity, acceleration, centrifugal force, spring force, damping force, inertia force, force vectors, vibration modes, impact test

measured by a sensor placed at a point on the bearing housing—occurs with little or no time lag after the moment the "heavy spot" passes the same point. We say the "high spot" and "heavy spot" are approximately in phase (see Figure 1).

As rotating speed increases and approaches the natural frequency of the system, two things happen. Firstly, vibration amplitude may increase significantly, peaking at the point where the rotating speed equals the natural frequency (a local maximum). Secondly, an increasing phase lag will develop—that is, a delay between the heavy spot passing a given point, and the peak of the measured vibration displacement at that same point. This lag will equal 90 degrees of shaft rotation when the rotating speed equals the system natural frequency.

As rotating speed is increased still further, amplitude will initially reduce (from the local maximum) and the phase lag will continue to increase, tending toward 180 degrees when the frequency of the unbalance force is well above the natural frequency.

This behavior can be demonstrated experimentally during machine run ups and coast downs, plotting vibration amplitude and phase against frequency on a graph know as a bode plot (an idealized example is shown at Figure 2).

To understand the mechanism that dictates this behavior, we must first understand two underlying principles of motion in an oscillating system:

- The (constant) *phase relationship* between displacement, velocity and acceleration, and,
- The (changing) *amplitude relationship* between displacement, velocity and acceleration . . . and how this is affected by frequency.

We will now consider each of these in turn.

ACCELERATION ALWAYS OPPOSES DISPLACEMENT

In any oscillating system, at any given moment, acceleration will always be in the opposite direction to displacement. This is fundamental to our explanation but may seem at first glance to be somewhat counterintuitive. Let's examine why it is true. For this we will consider the simpler case of a single degree of freedom system—a mass supported on a spring as shown in Figure 3 (see page 6)—experiencing damped free vibration. That is, we apply a single impulse force to the mass in the vertical direction and allow it to oscillate up and down.

We can plot a full cycle of displacement from the at-rest or reference position, as shown in the upper plot of Figure 4 (see page 6).

If we now examine its velocity, we see that the peaks of velocity occur as it passes through the reference point (zero displacement) in each direction, and that at the positive and negative peaks of displacement—at the extremes of movement—the velocity must be zero. The velocity cycle may thus be considered to lead the displacement cycle by 90 degrees, as shown in the center plot of Figure 4. Now considering the acceleration cycle, we note that the peak of negative acceleration is at the peak of positive displacement and vice versa, as seen in the lower plot of Figure 4.

Acceleration therefore leads the displacement by 180 degrees and leads velocity by 90 degrees. These phase relationships will always apply in any vibration, regardless of cause, amplitude, or frequency.

But the most important point to grasp here is that at the point of maximum displacement in either direction, the spring force will be imposing an accelerating force in the opposite direction—*acceleration always opposes displacement*.

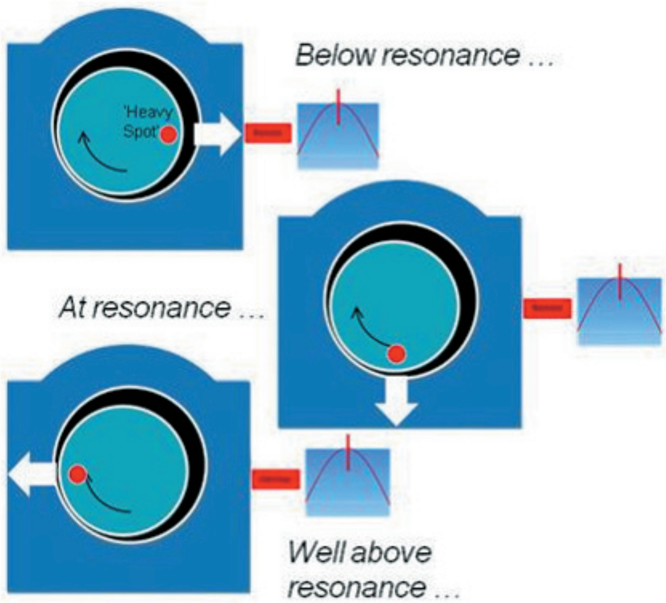

Fig. 1: Phase lag below, at, and above resonance

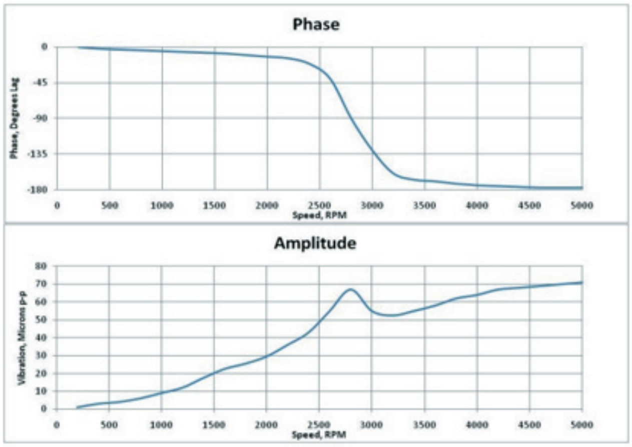

Fig. 2: Idealised bode plot of vibration amplitude and phase vs. speed

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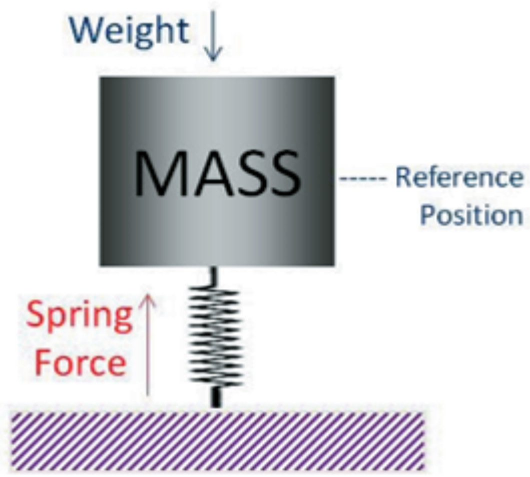

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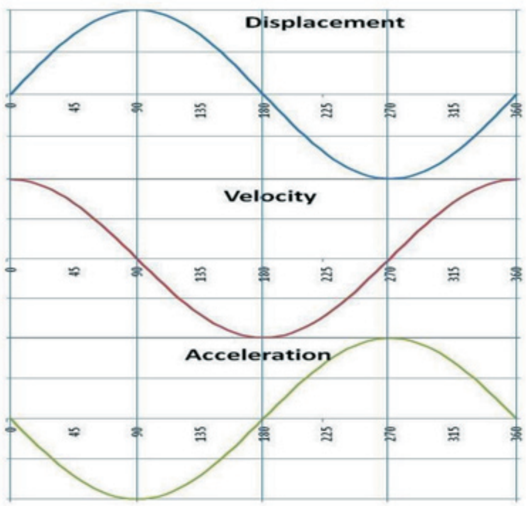

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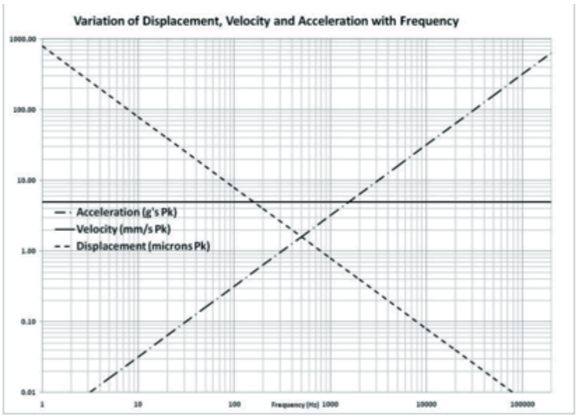

Fig. 3: Mass-spring system
Fig. 4: Displacement, velocity, acceleration—phase relationship
Fig. 5: Effect of frequency on displacement, velocity, and acceleration

ACCELERATION AND DISPLACEMENT AMPLITUDES ARE FREQUENCY DEPENDANT

If we were to choose some fixed amplitude of vibration velocity and could hold it constant but vary the frequency, we would find that increasing the frequency will increase acceleration amplitude whilst reducing displacement amplitude.

This can be reasonably inferred from the equations that relate these three parameters to each other. However, a more intuitive understanding comes of realizing that with a higher frequency, and thus shorter period, the mass has less time in which to travel and thus cannot travel so far—hence lesser displacement. Similarly, a shorter period in which to achieve a given peak velocity means the mass must experience much greater peak acceleration. This relationship is often represented in a graph similar to that at Figure 5.

In summary:

- At *low* frequencies, displacement amplitude is high, acceleration is low;
- At *high* frequencies, displacement amplitude is low, acceleration is high.

A TALE OF FOUR FORCES

Let us return to our turning rotor. The *main* forces acting upon it are:

1. Centrifugal Force (due to residual unbalance)
2. Spring Force (due to bearing and support stiffness—a reaction to displacement)
3. Damping Force (due to viscous shear of the oil film—a reaction to velocity)
4. Inertia Force (the reaction of the mass to acceleration)

Centrifugal Force (Fc) is the reaction force to angular acceleration. It acts in the direction of a line drawn from the center of rotation through the center of mass and is given by $Fc = Me\omega 2 = Me \text{ x } (2\pi n/60)2$, where M is the shaft mass in kilograms, e is the eccentricity (the distance between the center of rotation and center of mass) in meters, ω is the angular velocity in radians, and n is the turning speed in rpm.

Spring Force (Fs) is the restoring force created by the stiffness of the shaft, bearings and/or support when displaced from the "at rest" position. It will always directly oppose *displacement*. $Fs = kX$, where k is the spring constant in Newtons/Metre, and X is the displacement in meters. So,

- *Spring force is proportional to Displacement.*
- *Spring force opposes Displacement.*

Damping Force (Fd) is the force created by fluid shear when an object has velocity in that fluid, and it will directly oppose that *velocity*. $Fd = cV$ (or $c\omega X$); where c is the damping constant in Newton-seconds/meter and V is the vibration velocity in meters/sec. If you wade through water, you will feel damping force as a "drag" resisting your motion.

- Damping force is proportional to Velocity.
- Damping force opposes Velocity.

Inertia Force (Fi) is the reaction force produced when we try to accelerate any mass, and it will directly oppose *acceleration*. Fi = MA (or Mω2X), where M is the mass experiencing the acceleration and A is the acceleration. If you try to push a car on level ground, you will feel inertia force resist your efforts to get it moving.

- Inertia force is proportional to Acceleration.
- Inertia force opposes Acceleration.

We are now in a position to draw some important conclusions. If:
- Displacement and acceleration act in opposite directions and
- Spring force opposes displacement and
- Inertia force opposes acceleration

Then:
- *Spring force and inertia force must also oppose each other.*

Due to the amplitude relationship between displacement and acceleration amplitudes noted in Figure 5, as machine speed and excitation frequency increases, the acceleration (and thus inertia force) will increase more rapidly than displacement (and the associated spring force). Eventually, a point is reached where the spring and inertia forces become equal and negate each other, leaving damping force alone to limit vibration amplitudes. If damping in our machine is low, this means that there is little to restrain amplitudes until they become very high—that is, high enough for the damping force to equal the excitation force. The frequency at which this occurs is fn, the natural frequency.

This explains why there is a local maximum in the vibration amplitude at this point and why this maximum can be very significant if damping is low.

But what of the phase shift? To understand this, we need to resort to some basic vector diagrams.

RESOLUTION OF FORCES USING VECTORS

Let us consider how these forces might act on a rotating shaft, with a certain degree of unbalance force. If the shaft is rotating at steady speed and producing a stable level of vibration, it is reasonable to assume that all the forces, although rotating together, are in equilibrium (that is, their vector sum is zero). To understand how phase shifts occur, we shall examine three cases of how these forces balance:

- When excitation frequency is well below natural frequency,
- When excitation frequency equals the natural frequency,
- When excitation is well above natural frequency.

In the following vector diagrams, please note these are not to the same scale since the unbalance force will of course increase proportional to the square of the speed. The reader should, however, focus on the magnitudes of the forces relative to each other, since it is this that produces the phase shift.

To avoid cluttering the diagrams, please note that the curved arrow labelled ωt simply indicates the direction of rotation, and ωt is the instantaneous phase angle. The location of the theoretical "heavy spot" created by the offset center of mass would be at the tip of the vector labelled "excitation force." Note that, as discussed, the acceleration vector leads the velocity vector by 90 degrees and the displacement vector by 180 degrees.

Figure 6 is a vector diagram showing the resolution of forces when turning speed is well below natural frequency. Displacement is high relative to velocity and acceleration because the exciting frequency is low. Consequently, damping force and inertia force are relatively small compared with spring force, and almost all the excitation force produced is expended in overcoming spring force. Therefore, displacement lags behind the excitation force by only a small amount.

In Figure 7, speed has now been increased such that the turning speed equals the natural frequency. The acceleration term in the inertia force has increased to the point where inertia force and spring force are now equal. This means that the only force now opposing the excitation force is the damping force. This is an undesirable area to operate in, since if the machine design provides little in the way of damping, there is consequently little to constrain vibration amplitudes and very high vibration may result.

Note that the phase of the unbalance force has not changed—the heavy spot has not moved! Thus the other forces—and hence the displacement, velocity, and acceleration vectors—must realign themselves to maintain equilibrium. Hence, displacement will now lag the excitation force by exactly 90 degrees.

In Figure 8, speed has further increased to well above the natural frequency. The acceleration term in the inertia force means this force now dominates the vector diagram, with relatively small influence from spring and damping forces. Again we note the phase of the excitation force has not moved, but this force is now mostly expended in overcoming inertia force. Again, the force vectors must realign to achieve equilibrium, thus so too must displacement, velocity, and acceleration. Hence, displacement now lags the excitation force by an amount approaching 180 degrees.

We can therefore divide the frequency range into three regions (see Figure 9 on page 9):

1. The *Spring Controlled Region*, where amplitudes are limited by the dominant spring force
2. The *Mass Controlled Region*, where amplitudes are limited by the dominant inertia force
3. The *Damping Controlled Region*, where spring and inertia forces cancel, leaving damping force alone to limit vibration amplitudes.

A possible revised definition of resonance might then read as follows:

> *"The condition where a system or body is subjected to an oscillating force, at a frequency such that spring force and inertia force are equal and negate each other—the natural frequency—leaving damping force alone to limit vibration amplitudes."*

SYSTEM NATURAL FREQUENCY

This condition of equality of the spring and inertia forces also shows the way to the derivation of the natural frequency of a system. This derivation is shown in Figure 10.

We have now covered what happens at resonance and why and determined why a mass spring system has a natural frequency.

There are other questions to consider, such as why some systems appear to have more than one natural frequency. The simple answer to this is that whilst there is only one natural frequency for a given mass and stiffness, some systems have different stiffness in different planes (e.g. a horizontally mounted machine will be more stiff in the vertical plane than the horizontal) and thus a different stiffness is applicable to the calculation.

Additionally, shafts may also have different modes of vibration (bouncing, rocking and bending modes for example), and different parts of the machine may have different natural frequencies. Considered together, a machine train may have many natural frequencies in different areas and different planes. When we consider also that as the speed of a machine changes and the excitation forces pass through these natural frequencies, the behavior described above may not have even completed its transition through the regions relevant to one natural frequency, before behavior starts to be affected by another. Add to this the fact that excitation frequencies at other than turning speed, produced by phenomena such as vane pass frequency for example, also pass through these zones, analysis can become quite complex!

DIAGNOSING RESONANCE

Resonance should be suspected if any of the following characteristics are present:

- Vibration at a particular frequency is disproportionately higher in one plane than in other planes,

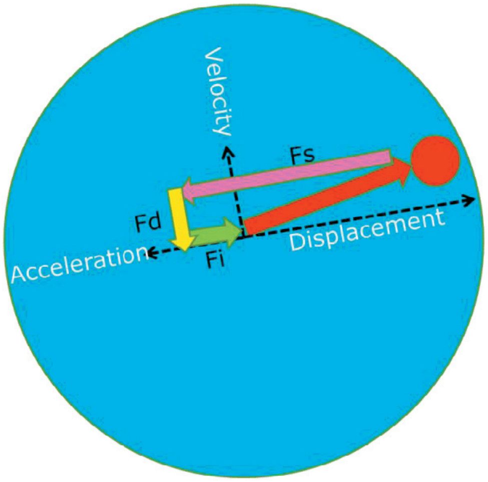

Fig. 6: Force vectors, speed well below natural frequency

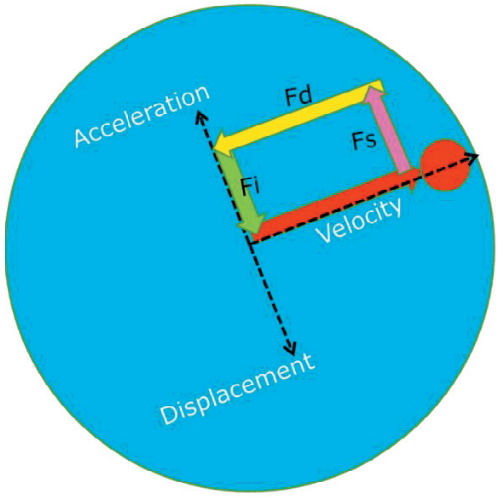

Fig. 7: Force vectors, speed equal to natural frequency

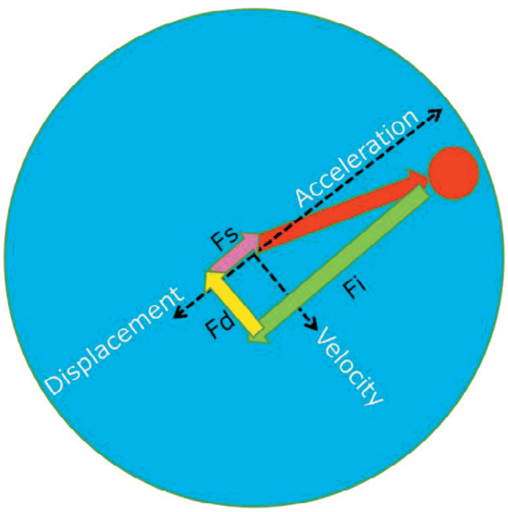

Fig. 8: Force vectors, speed well above natural frequency

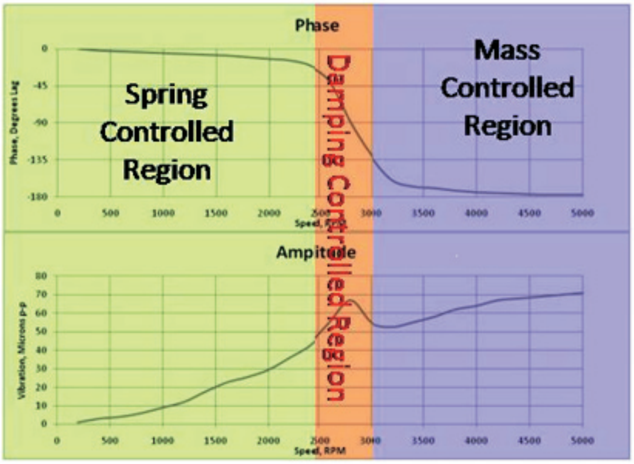

Fig. 9: Bode plot with overlay marking spring, damping, and mass controlled regions

As previously stated;

$$\text{Inertia Force } F_i = MA, \text{ and,} \quad (1)$$

$$\text{Spring Force } F_s = kD. \quad (2)$$

We know that

$$\text{Velocity } V = 2\pi f D, \quad (3)$$

and

$$\text{Acceleration } A = 2\pi f V \quad (4)$$

so substituting for V in (4) we obtain

$$A = (2\pi)^2 f^2 D. \quad (5)$$

At resonance, we know that

$$F_i = F_s, \text{ that is,}$$

$$MA = kD, \quad (6)$$

and, substituting for A in (6),

$$M(2\pi)^2 f_n^2 D = kD \quad (7)$$

The two Displacement terms cancel, and transposing to make f_n^2 the subject, we obtain

$$f_n^2 = \frac{k}{M(2\pi)^2}, \quad (8)$$

and, taking the square root of both sides,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{M}} \quad (9)$$

Fig. 10: Derivation of natural frequency formula (in Hertz)

- The design of the structure has lower stiffness in that plane,
- In a variable speed machine, the amplitude at a particular order of turning speed varies very significantly with speed,
- The shape of the peak may have an unusually wide base—but this is not nearly as reliable as the fault finding charts seem to suggest! It depends on the degree of damping.

To close, let us take a brief look at a simple case study.

A variable speed motor, mounted above a belt driven pump (Figure 11), was found to have very high vibration at motor turning speed in the horizontal (Figure 12, see page 10) but had very low amplitude at the same frequency in the vertical (Figure 13, see page 10).

Data acquired later at a slightly higher speed resulted in much lower amplitude (Figure 14, see page 10); a simple impact or "bump" test confirmed the diagnosis (Figure 15, see page 10).

Most such machines are usually designed so that the first natural frequency is well above likely excitation frequencies. Thus the first avenue to explore was to check that all was as intended with the support structure and that nothing had gone awry to reduce the stiffness below design—i.e. all fasteners were checked secure, and the frame inspected for cracked welds, enlarged holes, or other damage that could reduce stiffness. No such issues were found here, however.

A number of possible remedies were explored, including adding diagonal cross-bracing between the four threaded bar "legs" supporting the motor platform. However, it was found that the natural frequency could be moved somewhat higher by using shorter belts and reducing the length of the legs / height of the platform. Although this meant modifications to the belt guards, this was found to be a most effective solution.

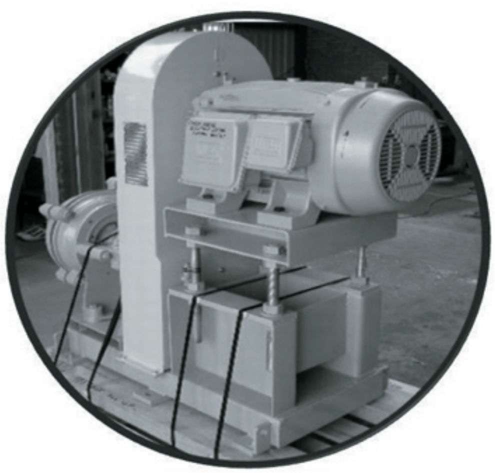

Fig. 11: Slurry pump, motor mounted on platform about pump

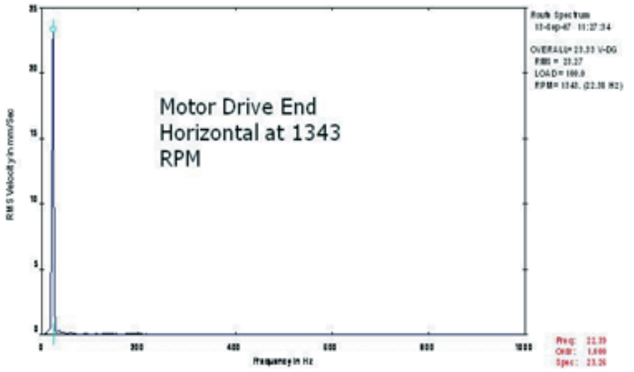

Fig. 12: Motor DE horizontal at 1,343 rpm

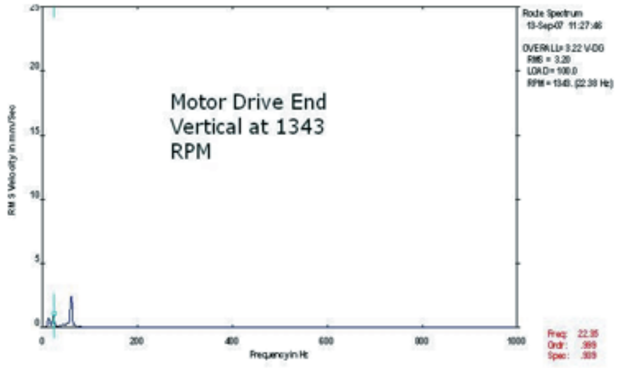

Fig. 13: Motor DE vertical at 1,343 rpm

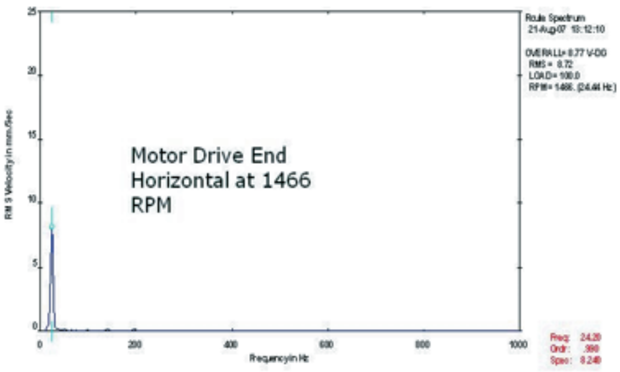

Fig. 14: Motor DE horizontal at 1,466 rpm

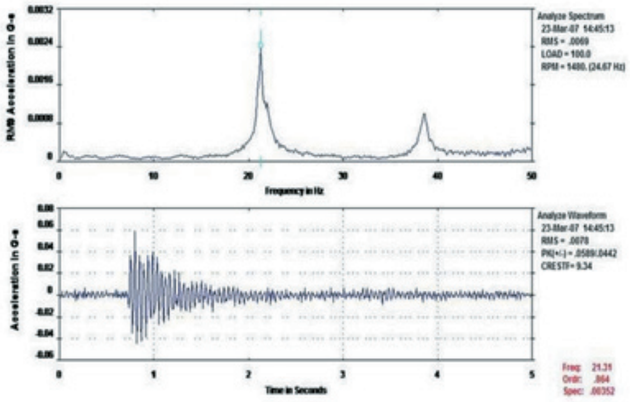

Fig. 15: Impact test results

Chris White *served for 20 years in nuclear submarines in the UK Navy, learning his trade as a mechanical and electrical technician and working his way to chief engineer; it was in this role he cut his teeth in vibration in 1987. He studied during the long patrols to gain an honors degree in software engineering, followed by a master's degree in mechatronics. On leaving the Navy in 2002, White specialized in predictive maintenance and has worked both with, and within, the mining, oil and gas, and utility industry sectors in developing and managing condition monitoring capabilities.*

White joined Wood (formerly SVT Engineering Consultants) in 2008 and continues to develop, including completing his Vibration Category 4 certifi cation with the US Vibration Institute in 2011. He divides his time between serving as technical authority within the Rotating Equipment Reliability business unit and delivering public vibration training courses. However, he would say that every day he is amazed and humbled by how much more there is to learn.