# THE INFLUENCE ON TORSIONAL VIBRATION ANALYSIS OF ELECTROMAGNETIC EFFECTS ACROSS AN INDUCTION MOTOR AIR GAP

E. G. Hauptmann, PhD, PEng, Director, Eng. Development, Lo-Rez Vibration Control.
W. F. Eckert, PhD, PEng, Principal Engineer, Beta Machinery Analysis.
B. C. Howes, MSc, PEng, Chief Engineer, Beta Machinery Analysis.

## ABSTRACT:

The effect of electromagnetic stiffness and damping on torsional vibration has been evaluated, and an approximate method for calculating the shift in torsional natural frequency due to ***em*** effects is presented. Systems with “very soft” couplings can be extremely sensitive to the ***em*** effect.

## INTRODUCTION:

AC polyphase induction motors produce rotating electromagnetic fields in the stator windings, which in turn induce currents in the rotor bars and thereby cause the rotor to turn and carry load under steady operation. If the rotor has a torsional vibration superimposed over the steady rotation, the same electromagnetic fields across the air gap between rotor bars and stator windings can produce torques acting on the rotor which can be considered to act as torsional springs and dampers. These additional electromagnetic (***em***) effects are not usually included in standard torsional vibration analysis, as no simple methods for estimating their magnitudes have been available to date.

Such unsteady ***em*** effects have nevertheless been extensively studied in the past by direct numerical integration of the differential equations representing stator and rotor currents and their mutually induced stator and rotor fields. Past studies have typically included startup of drive systems and estimation of the resulting transient motor torques. Jordan *et al* [1], [2], have developed the complete ***em*** field equations, which were then solved numerically showing dynamical effects such as limit cycles and drive instabilities (“negative” damping). Further work by Cierniak *et al* [3] extended this type of study to consider the effect of variable speed drives on dynamic current and slip characteristics.

As important as these past works are, the direct ***em*** effects on **torsional vibration** in a drive train have not been extensively explored. Most recently, Knop [4] presented a simplified approach to solving the ***em*** field equations by a “linearization” technique, and then showed how one could gain a better understanding of the importance of ***em*** effects by first referring to simpler mass – elastic models.

As an example, Figure 1 shows an idealized two-mass torsional model of a compressor and motor rotor, with the addition of an ***em*** “spring” and “damper” acting between the rotor and stator.

As is well known, the additional spring and damper add a second vibrational mode, so that the system now has two natural frequencies, one above and one below the single mode frequency. This effect was described by Knop [4] in a figure similar to Figure 2.

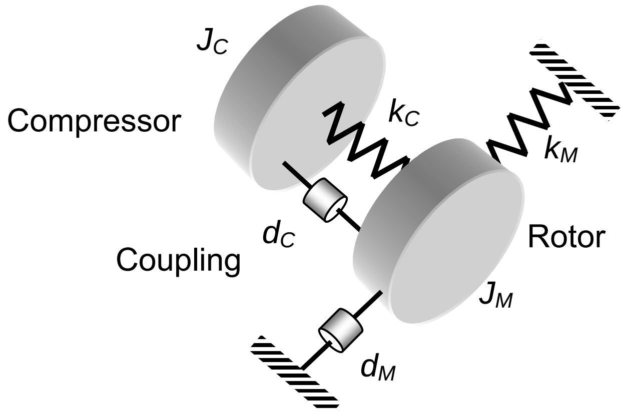


**Figure 1.** An idealized two-mass model of a motor-compressor drive, with an additional ***em*** spring and damper between the rotor and stator.

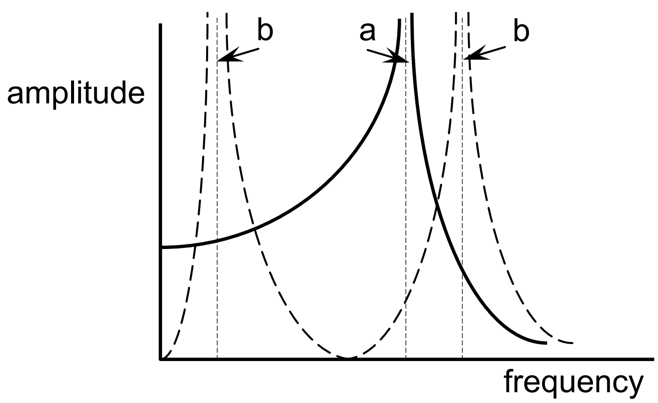


**Figure 2.** Torsional natural frequencies of a two-mass model: curve (**a**), single frequency, and no ***em*** effect; curves (**b**) including ***em*** effects, with the higher natural frequency possibly raised into the system operating range.

Knop [4] explained that since coupling stiffness's would normally be chosen to place the coupling mode below the operating speed range (heavy line **a** in Figure 2), the additional ***em*** spring could well raise the higher natural frequency into the operating range (the right hand dashed lines **b** in Figure 2). In Figure 3 (redrawn from Knop), Knop showed that for a coupling with stiffness lower than that of the ***em*** spring, $k_M / k_C$ , the natural frequency could be as much as 1.5 times higher than normally estimated!

The numerical examples given by Knop (ratio of rotor to compressor inertias less than 1.0; ratio of ***em*** stiffness to coupling stiffness greater than 1.0) are found in many European compressor-motor drivelines, but are not common in many North American installations. For example in a GMRC 2011 Case Study [5], the ratio of motor to compressor inertia was approximately 1.85 and the ratio of ***em*** spring to coupling stiffness was approximately 0.8. Extending the values in Figure 3, the natural frequency in this case would be raised by approximately 6%.

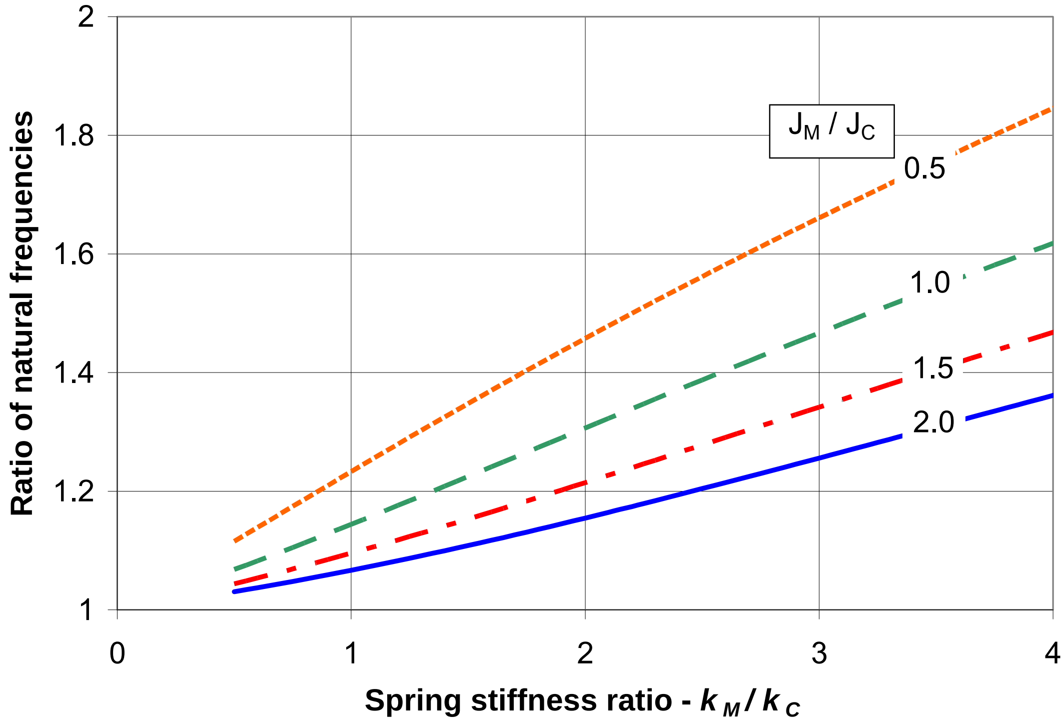


**Figure 3.** The effect of including an ***em*** spring in the two-mass model (neglecting damping, and taking $k_M$ as constant), taken from [4].

It must be emphasized that the above remarks are based on studying a simplified two-mass model, with a constant value of ***em*** spring stiffness and no accounting of damping effects. In the following we present a way to develop estimates of ***em*** spring and damping magnitudes, and consider their effect on torsional vibration response for a range of typical North American motor-compressor installations.

## ESTIMATING ELECTROMAGNETIC SPRING AND DAMPING VALUES:

The earlier reference cited, Knop [4], suggested that by linearization of the ***em*** field equations, analytical expressions could be generated for ***em*** spring and damping values. While this would not allow study of certain dynamical aspects (limit cycles, "negative" damping) described by Jordan [2], practical estimations of torsional response in a wide range of driveline installations might still be made.

Knop's Equations (5), (6) for the ***em*** spring stiffness $k_M$, and damping $d_M$ are:

$$k_M = (M_{St}/\Omega_s)\{(\omega^2 T_L) / [1 + (\omega T_L)^2]\},\ldots \textbf{Eq'n (1)},$$

$$d_M = (M_{St}/\Omega_s)\{1 / [1 + (\omega T_L)^2]\}\ldots. \textbf{Eq'n (2)}.$$

where: $\omega$ = frequency of the superimposed torsional vibration,
$\Omega_s$ = electrical supply frequency,
$M_{St}$ = a motor circuit constant with dimensions of torque,
$T_L$ = a motor circuit electrical constant with units of time.

However, no further information was given in [4] regarding how the expressions $M_{St}$ and $T_L$ could be evaluated, nor was further information made available to us. Knop was careful to point out that these equations could not produce "negative damping", and strictly speaking would be valid only for motors with negligible stator winding resistance. Knop also showed results for a particular case by reference once again to full numerical evaluation of the differential equations.

To uncover the meaning and get estimates for the variables $M_{St}$ and $T_L$ in Eq'ns (1) and (2), we have referred to the fundamental analysis and equations in Jordan *et al* [1], [2]. We have also carried out a further linearization analysis of expressions for spring and damping effects used in their studies. This involves algebraic manipulation not worthy of inclusion here. A brief outline of this analysis is given in the Appendix, and interested parties should contact the lead Author for details.

As outlined in the Appendix, we have derived the following results for $M_{St}$ and $T_L$:

with; $T_R$ = rated (full load) motor torque,
$T_B$ = breakdown motor torque,
$s_R$ = slip at rated load,
then: $T_L \ \mathbf{z} \ (1/\Omega_s)\{1/(2s_R)\}\{T_R / T_B\}\ldots.$ **Eq'n (3)**, and
$M_{St}/(\Omega_s T_L) \ \mathbf{z} \ (\#\text{ stator poles})(T_B)\ldots.$ **Eq'n (4)**.

Using Equations (3), (4) in Equations (1), (2) will allow ready estimations to be made of the ***em*** effect on torsional vibration in a range of drive installations.

With the further substitution of: $x = (\omega T_L)$ , Eq'ns (1) and (2) become,

$$k_M = (\#\text{ stator poles})(T_B)\{x^2 / [1 + x^2]\},\ldots \textbf{Eq'n (5)}, \text{ and}$$

$$d_M = k_M / (\omega^2 T_L) = k_M (T_L) / (x)^2 \ldots. \textbf{Eq'n (6)}.$$

While the main motor torque characteristics $T_B$ , $T_R$ and $s_R$ are tabulated and readily available, the time constant $T_L$ requires a further evaluation of Eq'n (3). Figure 4 shows estimates made using Eq'n (3) for two types of motors, with time constants $T_L$ ranging from 0.07 to 0.14 seconds in the range of 500 to 1,500 kW. With torsional frequencies, $\omega$ , of interest from 10 – 60 Hz, the variable $x^2 = (\omega T_L)^2$

can vary from approximately 20 to 2,000, but the quantity [ $x^2/(1+x^2)$ ] in Eq'n (5) is always within a few percent of 1.0 and in such cases can be ignored. *However for very soft couplings between large motor-compressor setups, the frequency effect on kM shown in Eq'n 5 can be significant and should not be neglected.*

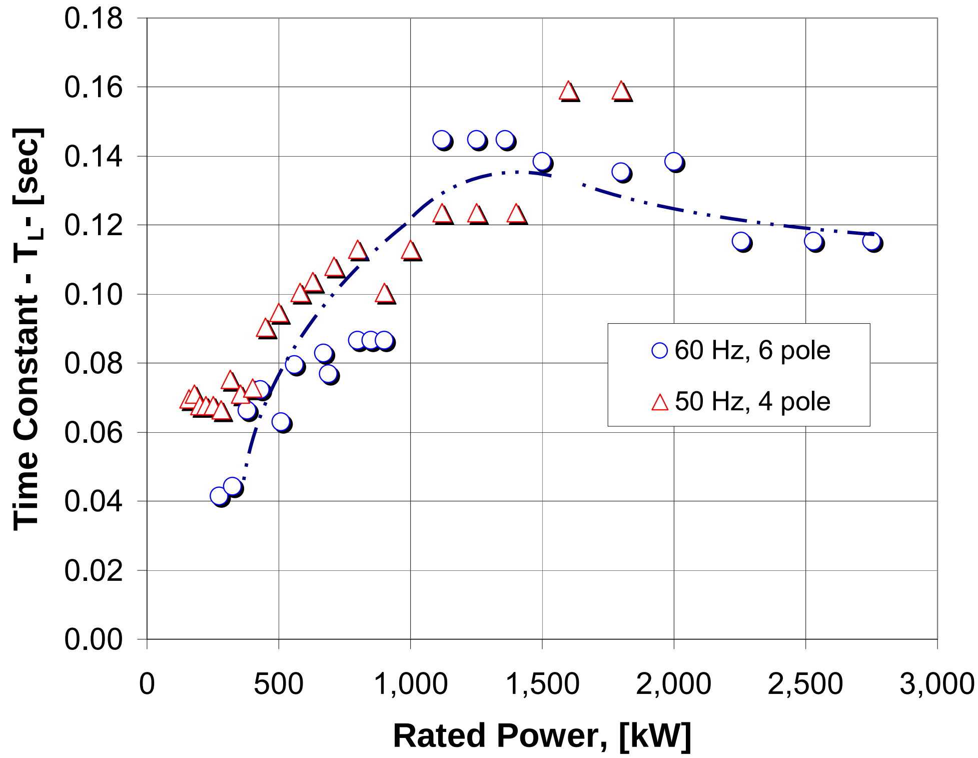


**Figure 4.** Estimated time constant $T_L$ for typical induction motors; data taken from published information [6], [7]. The dashed line is for reference only.

Before study of some previous installations, we once again consider the simple two-mass model shown in Figure 1. While it is clear that ***em*** damping must always be present along with an ***em*** spring, in most cases damping does not strongly influence estimates of *coupling mode* natural frequencies, and so can be neglected in this simple model. The calculation of natural frequencies for the two-mass model is made more difficult with a frequency-dependent ***em*** spring, but can be reduced to finding the roots of a quadratic equation whose constants are functions of $M_{St}$ and $T_L$ in addition to the constants without an ***em*** spring.

As an example; in the previously cited Case Study [5], taking the value of $J_M/J_C$ of 1.85 as before, with the time constant $T_L$ approximately = 0.12 sec., $M_{St}/(\Omega_s T_L)$ is approximately = 99 kNm, the coupling stiffness $k_C$ = 124 kNm/rad, so that the abscissa in Figure 5 is approximately = 0.8. Figure 5 shows that for this case, the first natural frequency is once again increased by about 7% as in Figure 3, but now the ***em*** spring stiffness has been evaluated using Eq'n (5). *Subsequent field* *measurements on this installation confirmed that the 1st TNF was about 8% higher than predicted by the original TVA, which did not include an* ***em*** *spring.*

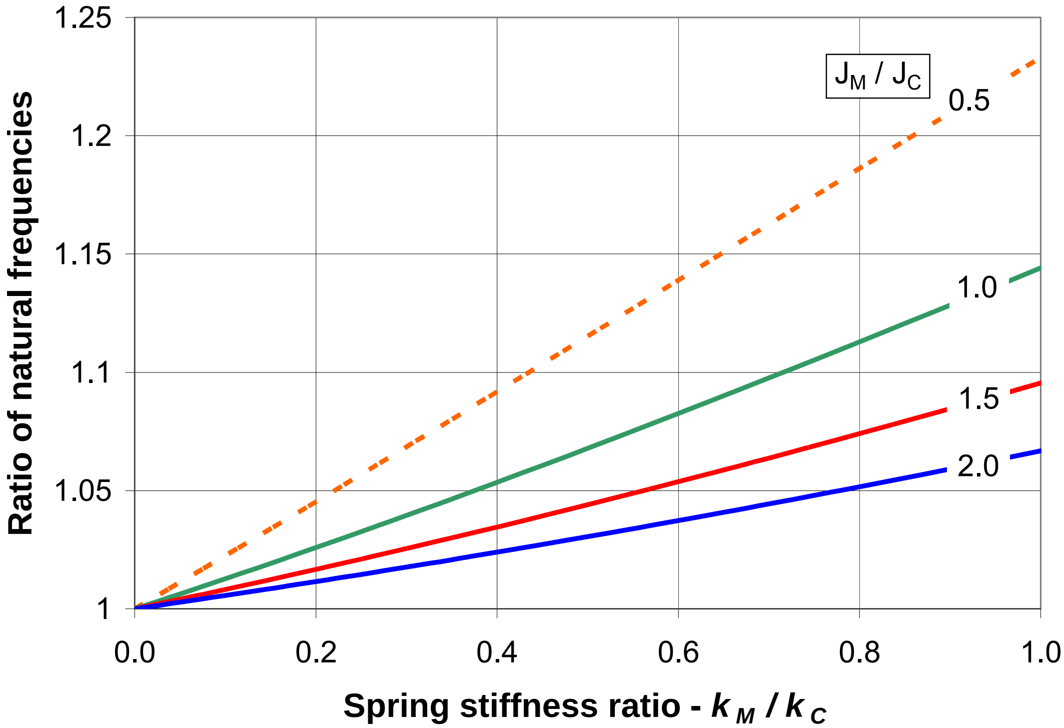


**Figure 5.** Results for the higher torsional natural frequency of the two-mass model, compared to that with no ***em*** effect, using Eq'n (5) to evaluate $k_M$ .

**CASE STUDIES:**

Equations (5) and (6) have been used to make estimates of $k_M$ and $d_M$ for a series of past torsional vibration studies in order to determine how important ***em*** effects may be. A representative sample of these studies is shown in Table 1, which summarizes the results for three types of installation:

1. Drive A; high power (5,000 HP), soft (rubber) coupling, (450-900 RPM),
2. Drive B; mid-range power (1,250 HP), steel spring coupling, (1,200 RPM),
3. Drive C; low power (450 HP), stiff coupling, mid-speed (720 RPM).

**(a) Results of *em* effects on natural frequency (coupling mode).**

Table 1 indicates that for systems like C, that is, $k_M << k_C$ , ***em*** effects are relatively unimportant. On the other hand systems like A, $k_M >> k_C$ , the coupling mode frequency can be as much as 90% higher, much as predicted by Knop [4]. The variable stiffness characteristics of rubber couplings are a particularly important influence on the wide range of frequency estimates for this type of system. Intermediate systems like B are modestly affected. These results are also shown in Figure 6, where $k_M$ in the abscissa is calculated from Eq'n (5).

| | Drive A | | | Drive B | Drive C |
|---|---|---|---|---|---|
| **Compressor** | Ariel JGC/6 | | | Ariel JGK/4 | Cameron MI-164 |
| speed range-RPM | 450-900 | | | 1185-1192 | 716 |
| | | | | | |
| **Coupling** | soft; rubber-in shear | | | steel-spring | stiff: disc pack |
| | cold | nominal | Warm | | |
| $k_C$ – kNm/rad | 136 | 106 | 72 | 124 | 15,117 |
| 1st TNF, $\omega_0$ - Hz | 5.58 | 4.93 | 4.04 | 14.75 | 77.98 |
| | | | | | |
| **Motor** | Toshiba | | | Reliance | Westinghouse |
| rated power - kW | 3,728 | | | 900 | 336 |
| line frequency - Hz | 60 | | | 60 | 60 |
| # poles | 8 | | | 6 | 10 |
| rated torque, $T_R$ - Nm | 40,230 | | | 7,210 | 4,474 |
| rated slip, $s_R$ - % | 1.67 | | | 0.67 | 0.56 |
| brk.dwn torq, $T_B$ - Nm | 88,507 | | | 16,593 | 8,948 |
| | | | | | |
| time const, $T_L$ – sec. | 0.036 | | | 0.086 | 0.118 |
| $x_E = (\omega_E T_L)$* | 3.40 | 3.40 | 3.40 | 10.8 | 8.88 |
| $x_E^2 / (1 + x_E^2)$ | 0.92 | 0.92 | 0.92 | 0.991 | 0.987 |
| $k_M$ – kNm / rad | 651.8 | 651.8 | 651.8 | 98.7 | 88.4 |
| $d_M$ – kNms / rad | 2.03 | 2.03 | 2.03 | 0.07 | 0.133 |
| $k_M / k_C$ | 4.79 | 6.15 | 9.05 | 0.80 | 0.0058 |
| $\Omega / \omega_0$ | 1.55 | 1.66 | 1.86 | 1.089 | 1.002 |
| * main excitation harmonic assumed to be 1 x run speed; $\omega_E$ = 94.25 rad/s for drive A | | | | | |

**Table 1.** A summary of ***em*** effects on some previous torsional vibration studies. The effect is pronounced for higher power installations with ultra-soft rubber couplings, while lower power drive trains with torsionally-stiff couplings are not greatly affected.

Table 2 shows a summary of the torsional natural frequencies compared to the orders of run speed. As previously noted the largest shift in torsional natural frequency was for the Ariel JGC/6 with a very soft rubber coupling; the natural frequency was up to 1.86 times that without the ***em*** effects in the model. For this case, the torsional natural frequency was shifted enough to result in resonance at 1 x shaft speed at the lower end of the run speed range.

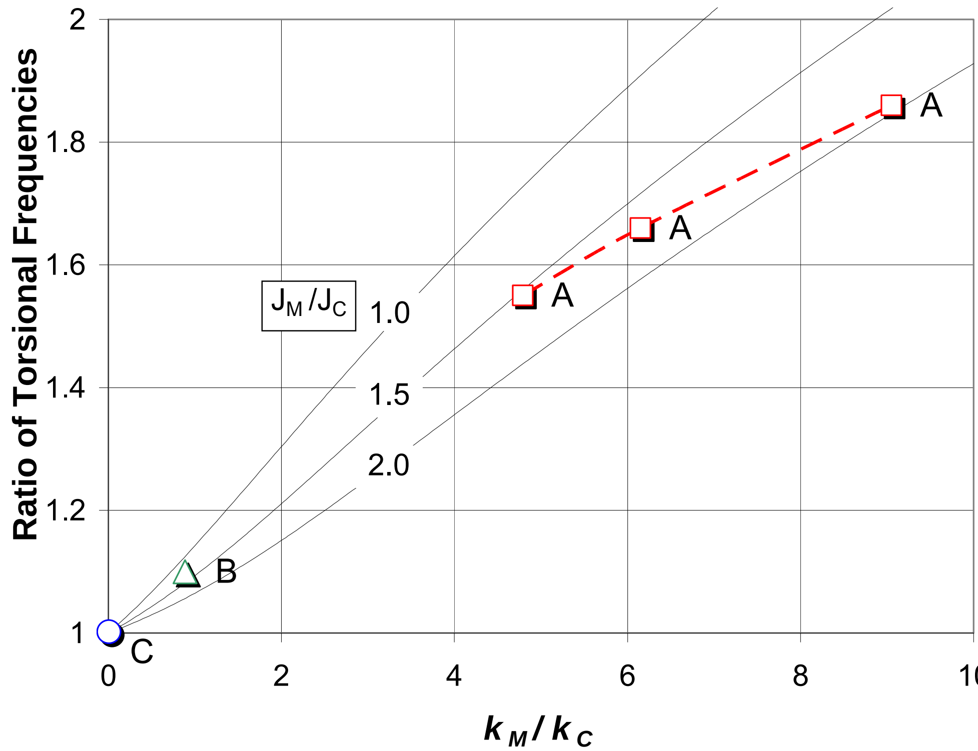


**Figure 6.** Representative ***em*** effects for the three different types of drives described in Table 1. The inertia ratio lines are taken from the two-mass model and are shown for reference only.

| predicted natural frequencies (Hz) | orders of run speed at | |
|---|---|---|
| | 450 rpm | 900 rpm |
| *without **em** stiffness* | | |
| 4.04 (coupling mode) | 0.54 | 0.27 |
| ***with em** stiffness* | | |
| 2.61 (rigid body mode) | 0.35 | 0.17 |
| 7.51 (coupling mode) | 1.00 | 0.50 |

**Table 2.** A summary of ***em*** effects on the predicted torsional natural frequencies of drive A (soft rubber coupling). The addition of the ***em*** stiffness results in the coupling mode being resonant at 1 x run speed.

### (b) Results of *em* effects on system stress and torque levels.

For each of the previous design cases A, B, C, the predicted torques, stress design factors, and coupling heat loads did not change to any significant degree during design operating conditions. This is because, although the system is now resonant with the coupling mode at run speed, there is very little 1 x torque demand from the compressor due to the staging and double acting loading for the given design operating conditions. However, if single acting conditions are considered (possibly as a result of a cylinder valve failure), the coupling vibratory torques and heat loads are predicted to be much higher when including ***em*** effects. In this case, the coupling heat load is approaching the manufacturer's limit during the upset conditions.

Table 3 shows a comparison of the predicted torques, design factors and heat loads. The added electromagnetic stiffness does have the potential to cause a resonant condition with resulting failures of driveline components during upset operating conditions. *At this point, the Authors are not aware of any actual cases where that has occurred.*

| | **compressor vibratory torque; $lb_f$-in** | **coupling vibratory torque; $lb_f$-in** | **coupling heat load (HP)** | **motor shaft stress design (Bagci) factor** |
|---|---|---|---|---|
| *design operating conditions* | | | | |
| no ***em*** effect | 516,900 | 28,720 | 0.77 | 51.5 |
| **with *em*** effect | 516,730 | 27,090 | 0.71 | 54.2 |
| *upset operating conditions (single acting cylinders)* | | | | |
| no ***em*** effect | 512,440 | 77,500 | 3.19 | 11.3 |
| **with *em*** effect | 513,300 | 125,980 | 4.67 | 15.3 |

**Table 3.** A summary of ***em*** effects on the predicted response of drive A (soft rubber coupling). In this case, the addition of the ***em*** stiffness and damping was insignificant for the design operating conditions, but important for upset conditions (particularly on the coupling vibratory torques and heat load).

## CONCLUSIONS:

The electromagnetic stiffness and damping effects on torsional vibration can be estimated by Equations (5) and (6). The predictions agree reasonably well with field measurements in a few cases known to us.

Systems with "very soft" couplings can be extremely sensitive to the ***em*** effect, particularly during upset conditions. Steel spring couplings may have their coupling mode frequency shifted upward by 8-10%, while "stiff" couplings are not significantly affected.

Reasonable preliminary estimates of the importance of ***em*** effects on a particular system can be made by taking the following steps:

1. Combine the inertias on either side of the coupling to form a simple two-mass model and note the inertia ratio.
2. Evaluate the abscissa in Figure 5, that is; # poles x breakdown torque / coupling stiffness.
3. Enter these values on Figure 6 to find how much the natural frequency might be affected. This is a very conservative estimate; very soft couplings might have much greater frequency shifts and in those cases, calculate $k_M$ from Eq'n (5).

Further field measurement of the coupling mode natural frequency for fixed speed induction motors is needed to confirm the validity of the approach used to develop Equations (5) and (6). Such field work requires advanced measurement and data processing techniques. The Authors plan to carry out such work, which will be reported in a future paper.

## ACKNOWLEDGEMENTS:

Andrew Mancini; for his helpful work in rerunning past analysis. Fabian Claussen, for translating several references from the original German to English.

## REFERENCES:

1. H. Jordan, J. Müller, H.O. Seinsch; Über elektromagnetische und mechanische Ausgleichvorgänge bei Drehstromantrieben; Wiss. Ber. AEG-TELEFUNKEN 53 (1979) 5.
2. H. Jordan, J. Müller, H.O. Seinsch; Über das Verhalten von Drehstromasynchronmotoren in drehelastischen Antrieben; Wiss. Ber. AEG-TELEFUNKEN 53 (1980) 3.
3. Cierniak, Funke, Wendt, Optimizing the drive system for variable speed electric motor driven reciprocating compressors, EFRC, Wien (2003).
4. G. Knop, The importance of motor dynamics in reciprocating compressor drives, EFRC, Düsseldorf (2012).
5. W.Jani, E.Hauptmann, Case Study: Tuning out a difficult torsional vibration problem in a reciprocating compressor installation, GMRC, Nashville, (2011).
6. Siemens, Three-phase induction motors, Catalog D 84.1 (2009).
7. ABB HV induction motors, technical catalog for IEC motors EN 09-2011-3

## APPENDIX:

List of symbols:

$\omega$ = angular frequency of torsional vibration
$\Omega_S$ = line frequency
$Z$ = $\omega / \Omega_S$
$p$ = no. of pole pairs; i.e. $2p$ is the no. of poles
$L_1$ = rotor leakage inductance
$L_2$ = stator leakage inductance
$I_M$ = rotor current
$R_1$ = stator resistance (per phase)
$R_2$ = rotor resistance (per phase)
$\alpha$ = $R_1 / \Omega_S L_1$
$\beta$ = $R_2 / \Omega_S L_2$

$\sigma$ = $1 - L_{12}L_{21}/L_1L_2$ = total scattering coefficient
a = $\beta^2(1+\alpha^2)$
b = $2\beta(\alpha\sigma - \alpha - \beta)$
c = $(\alpha+\beta)^2 + \sigma(\sigma - 2\alpha\beta)$
d = $\beta(\beta + 2\alpha s)$
s = slip
$s_R$ = slip at rated torque
$\gamma$ = $T_B / T_R$ = breakdown torque / rated torque

Eq'n (50), Jordan *et al* [1];

$$c_e = MZ\{ N_1/D_1 + N_2/D_2 \}, \text{ with } M = [3p^2L_1(1-\sigma)I^2_M]/[2(1+\alpha^2)];$$

$$N_1 = -(1+Z)[\alpha\beta - \sigma Z(1+Z)] + \alpha[\alpha Z + \beta(1+Z)].\ D_1 = [\alpha\beta - \sigma Z(1+Z)]^2 + [\alpha Z + (1+Z)]^2.$$

$N_2$ and $D_2$ have similar algebraic expressions, not shown here for brevity. Linearization involves of expansion of Z, limited to order ($Z^2$) with the result;

$$c_e = MZ^2(\sigma + \alpha^2)\{2a / [a^2 + Z^2(2ac-b^2)]\} \text{....}\textbf{Eq'n (A1)}.$$

The "electromagnetic" damping, $d_e$, is taken from [1], Eq'n (51), and as with $c_e$;

$$d_e = (M/\Omega_s)\{ 2ad / [a^2 + Z^2(2ac-b^2)] \} \text{....}\textbf{Eq'n (A2),}$$

with the constants a, b, c, d being algebraic combinations of $\alpha$, $\beta$, and $\sigma$.

From Knop, $\boldsymbol{k_M} / \boldsymbol{d_M} = \omega^2 T_L$, so that $\boldsymbol{k_M} / \boldsymbol{d_M} = c_e / d_e = \omega^2[(\sigma+\alpha^2)/(d\Omega_s)] = \omega^2 T_L$.

where $d = \beta(1+\alpha^2)$. Since $\alpha^2 << \sigma < 1$, from [1] we get;

$$T_L = (\sigma/\beta)/\Omega_s \text{ ...}\textbf{Eq'n (A3)}.$$

Further manipulation of Eq'n (A1) results in $M_{St} = 2M/\beta$... **Eq'n (A4)**.

To finally obtain useful expressions for $M_{St}$ and $\sigma/\beta$; from [1], Eq'n (50),

$M = [3p^2L_1(1-\sigma)I^2_M]/[2(1+\alpha^2)]$, also, Eq'n (26), $T_B = (3p(1-\sigma)L_1I^2_M)/2\sigma$,

so that with we get $M_{St}/\Omega_s T_L = 2pT_B$ = # poles x $T_B$ ... **Eq'n (A5)**.

To evaluate $\sigma/\beta$ : from Jordan [2], Eq'n (36); the rated motor torque, $T_R$ is given as

$$T_R = (3p(1-\sigma)L_1I^2_M)(\beta\sigma) / [(\alpha\beta - \sigma s_R)^2 + (\beta + \alpha s_R)^2].$$

So $T_B / T_R = \gamma = [(\sigma s_R)^2 + \beta^2] / 2\beta s_R\sigma$. Solving; $\sigma$ (neg. root) = $(\beta\gamma/s_R)(1/2\gamma^2 - 1/8\,\gamma^4 + \ldots)$,

or, $\sigma/\beta \approx 1/2\gamma s_R = (1/2s_R)(T_R / T_B) = T_L\Omega_s$ ....**Eq'n (A6)**.